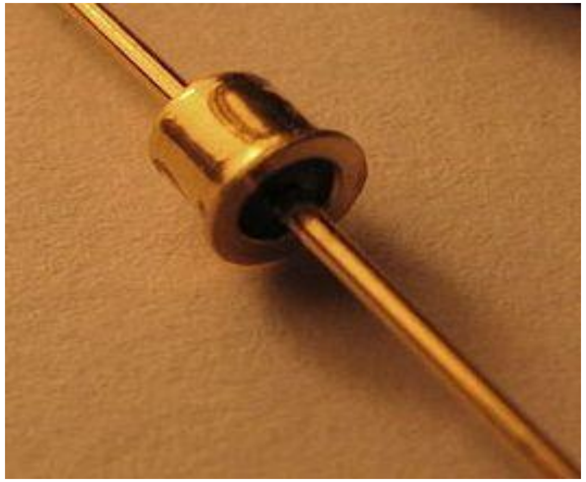
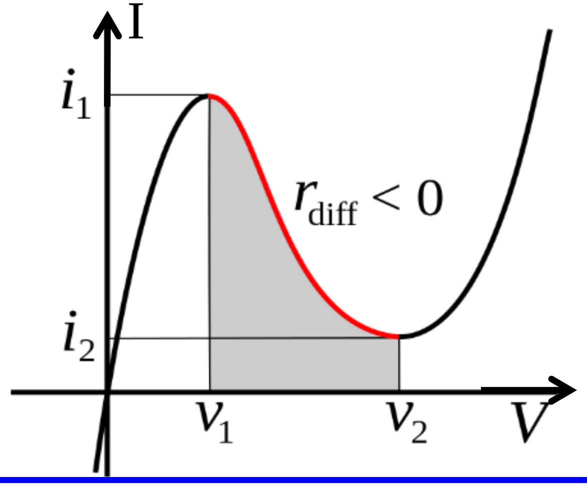
10 Things
You Didn't Know
About
Memristors

The *Perfect Storm* in Nonlinear Circuit Theory !

Esaki Diode

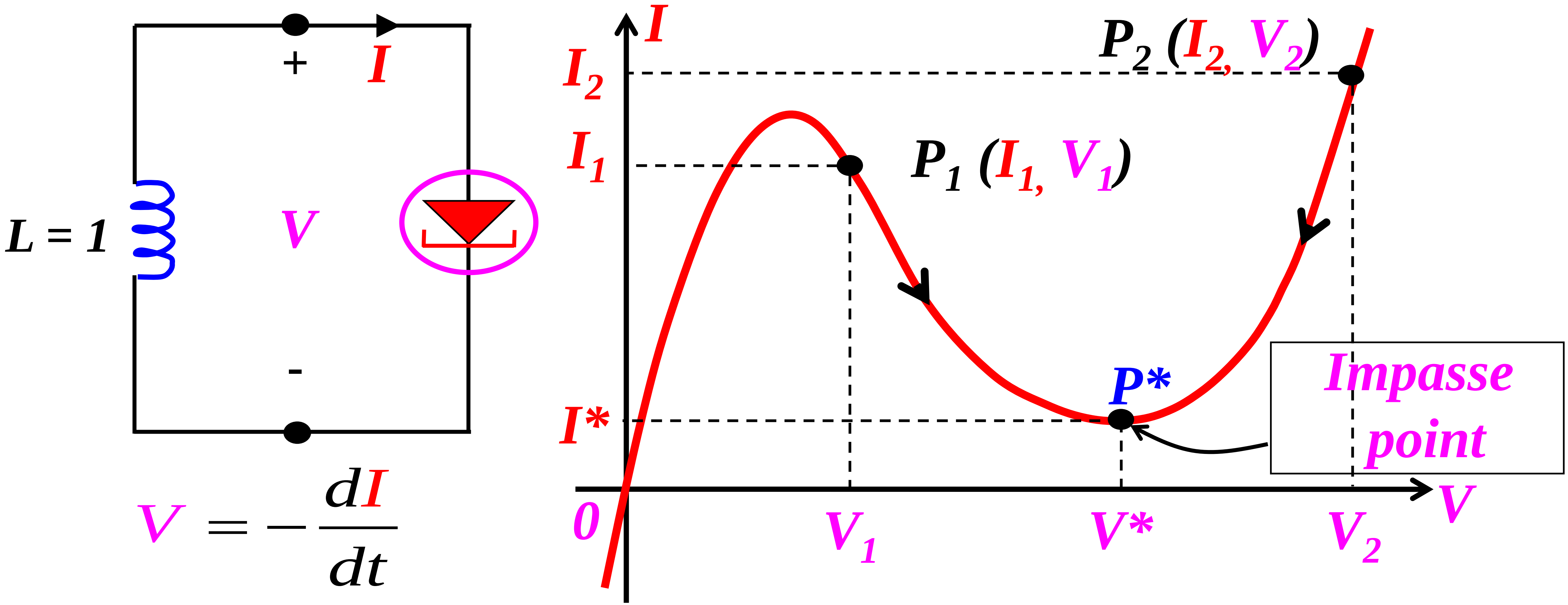




Leo Esaki
Nobel Prize in Physics, 1973

L. Esaki
New Phenomenon in Narrow Germanium p-n junctions
Physics review 109(2):603, 1958

Simplest *Tunnel Diode* Circuit



$$V = -\frac{dI}{dt}$$

Solution does NOT exist beyond P* !

Two Points of View !

- For ***mathematician***,

no solution is a

Perfectly ***valid solution***

- For ***everybody else***,

no solution means ***nonsence***.

Crisis in Circuit Theory

Pre-1970 Definitions of the

3 Basic Circuit Elements

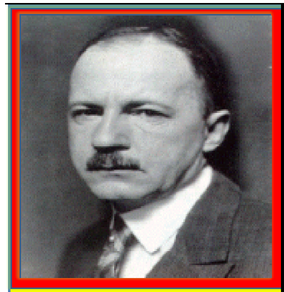
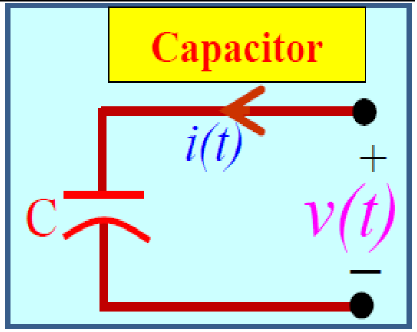
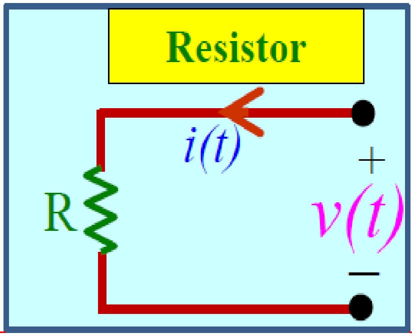
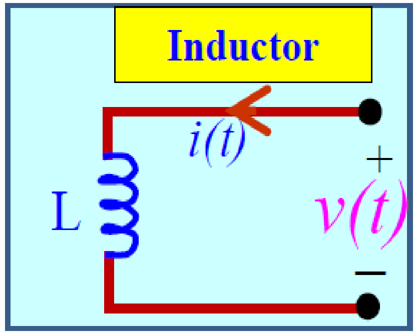
Capacitors, Resistors, and Inductors

give wrong circuit solutions

when the elements are

time-varying or nonlinear

3 Basic Circuit Elements

1745	 Ewald Georg Von Kleist		$$i(t) = C\,\frac{dv(t)}{dt}$$
1827	 Georg Ohm		$$v(t) = R\,i(t)$$
1831	 Michael Faraday		$$v(t) = L\,\frac{di(t)}{dt}$$

To Recover from

the perfect storm

Capacitors, Resistors, Inductors

must be

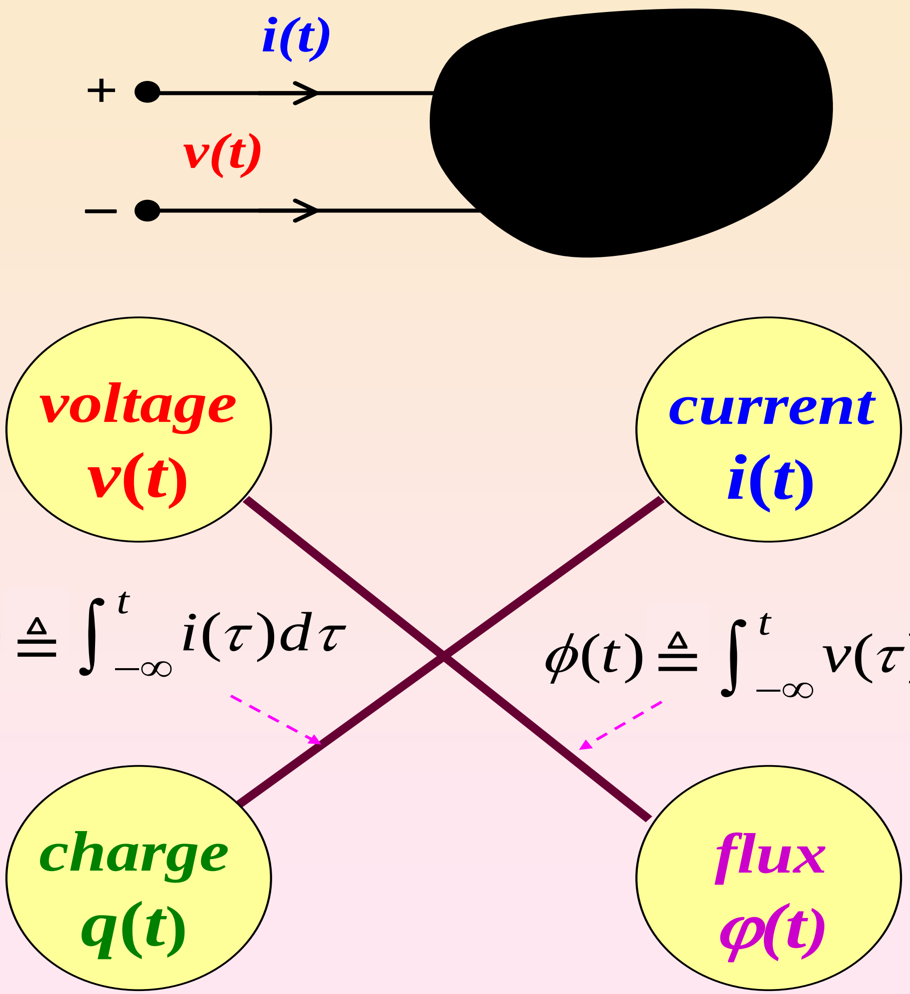
redefined

via an

AXIOMATIC APPROACH

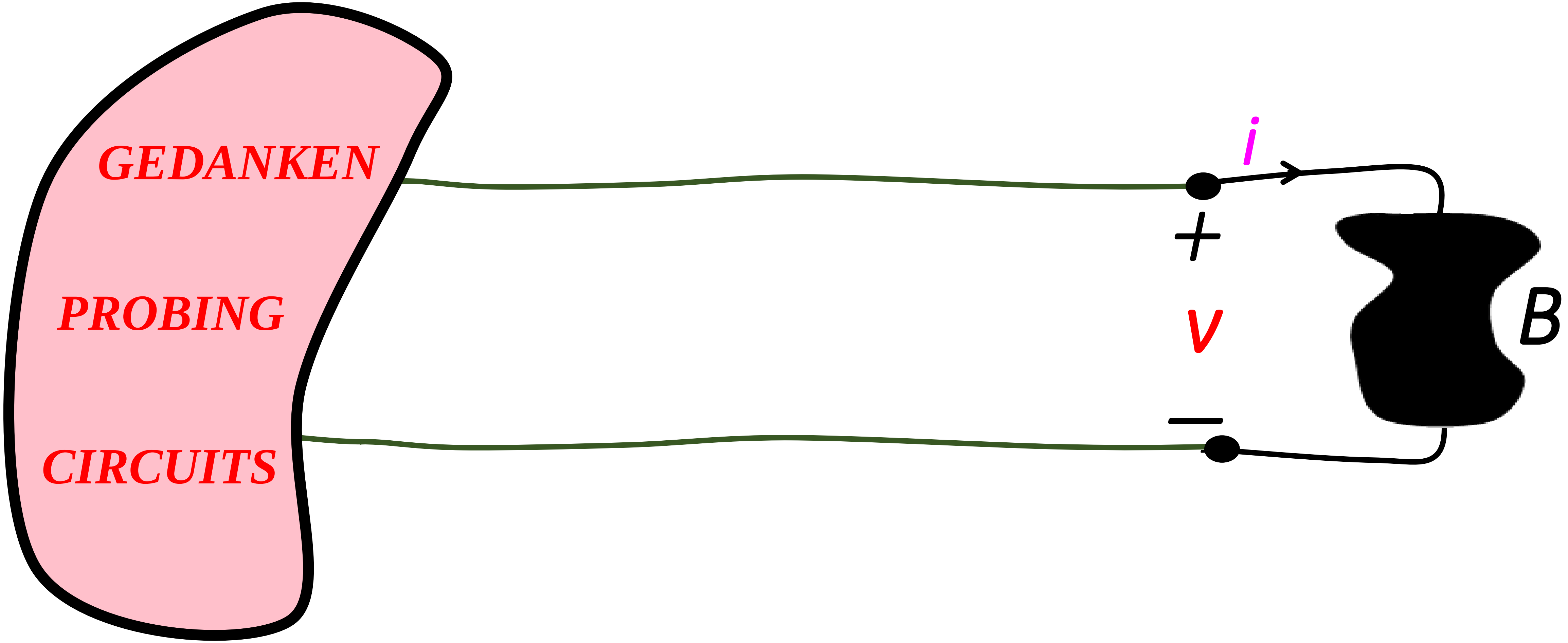
All Results Derived from An Axiomatic Approach are Timeless !

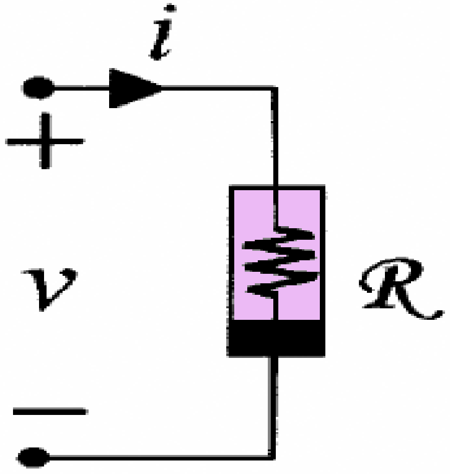
Four Basic Circuit Variables



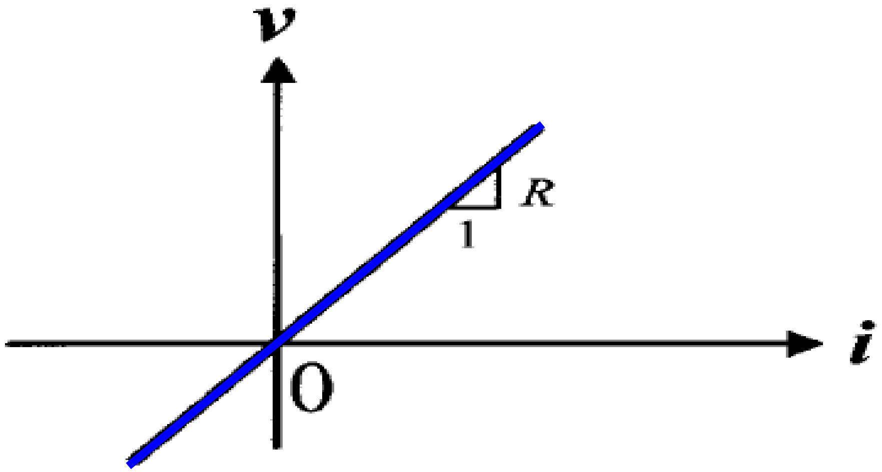
$$q(t) \triangleq \int_{-\infty}^{t} i(\tau)\,d\tau$$

$$\phi(t) \triangleq \int_{-\infty}^{t} v(\tau)\,d\tau$$


GEDANKEN
PROBING
CIRCUITS
i
+
v
−
B

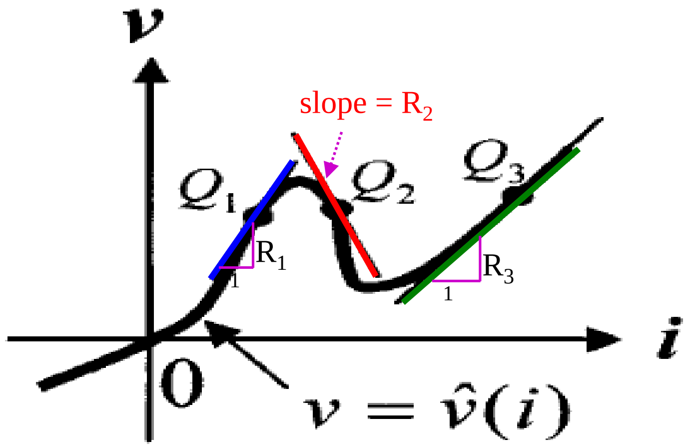


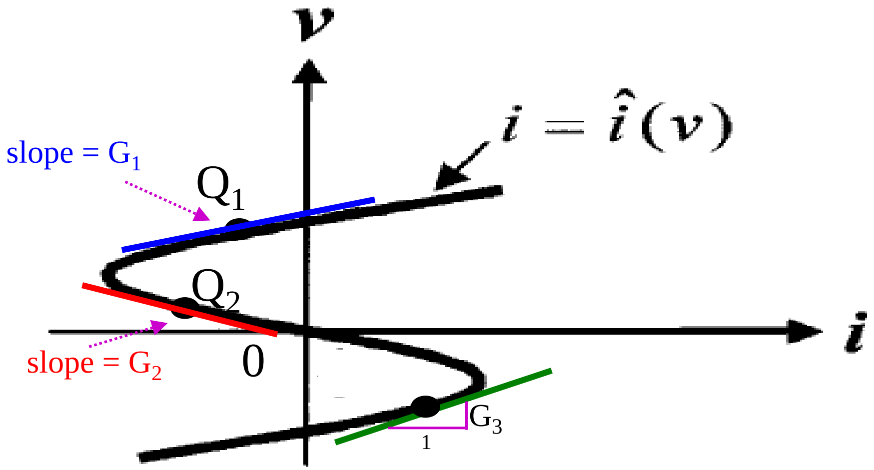
Nonlinear Resistor R



Linear resistor: $v = Ri$ or $i = Gv$

R = Resistance, $G \cong \frac{1}{R}$ = Conductance



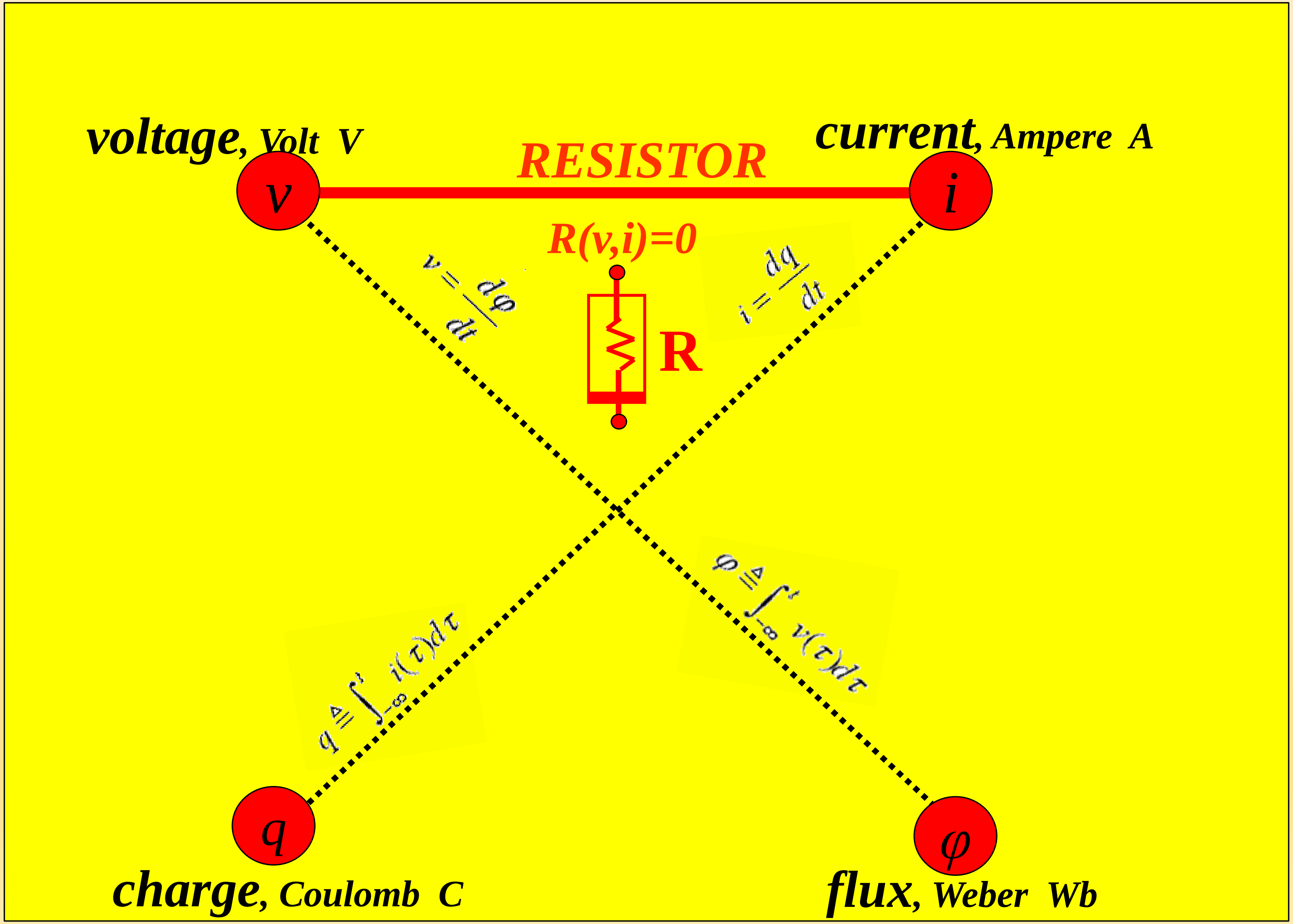


Current-controlled Resistor: $v = \hat{v}(i)$

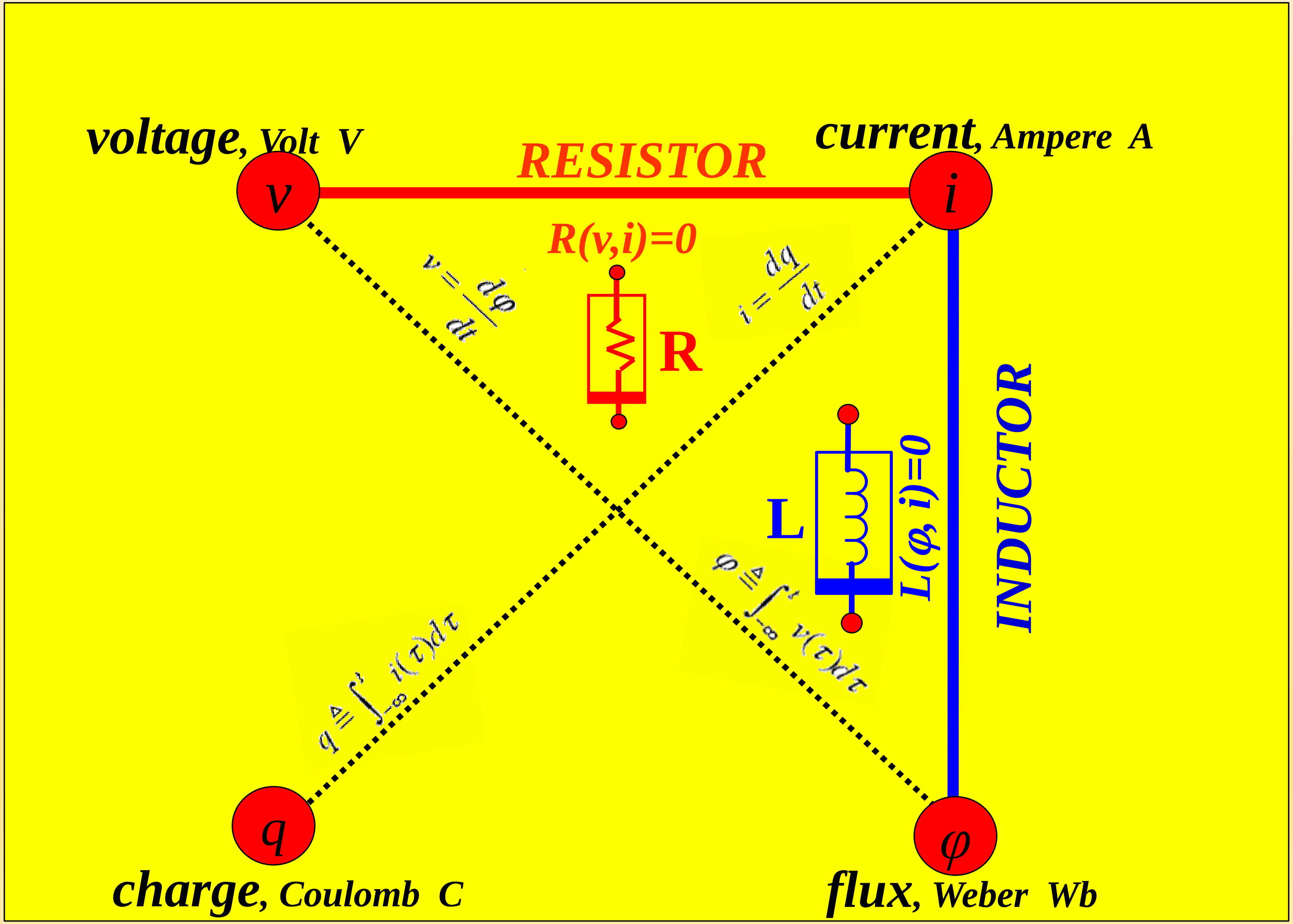
R_i = *small-signal resistance* at Q_i

Voltage-controlled Resistor: $i = \hat{i}(v)$

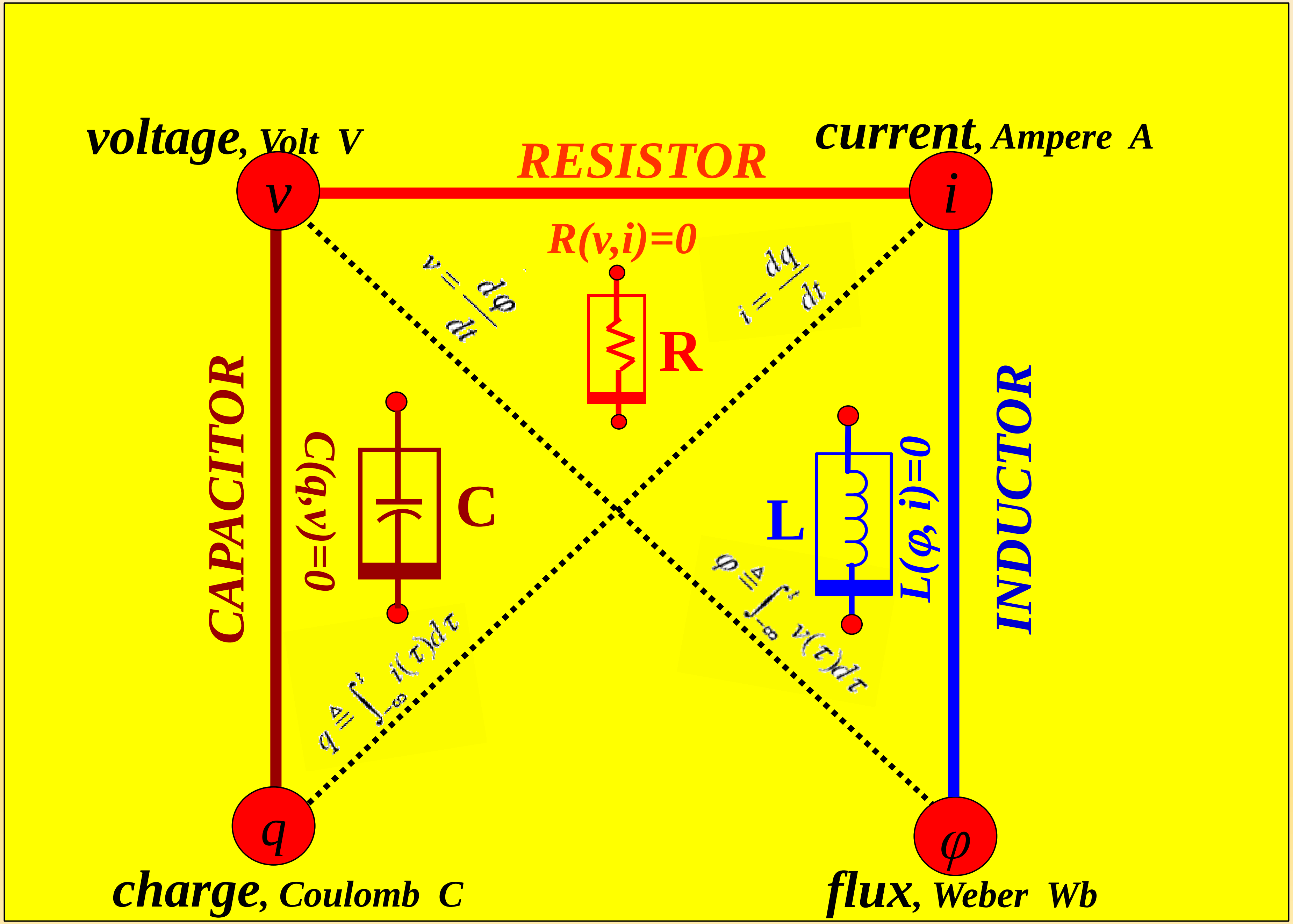
G_i = *small-signal conductance* at Q_i



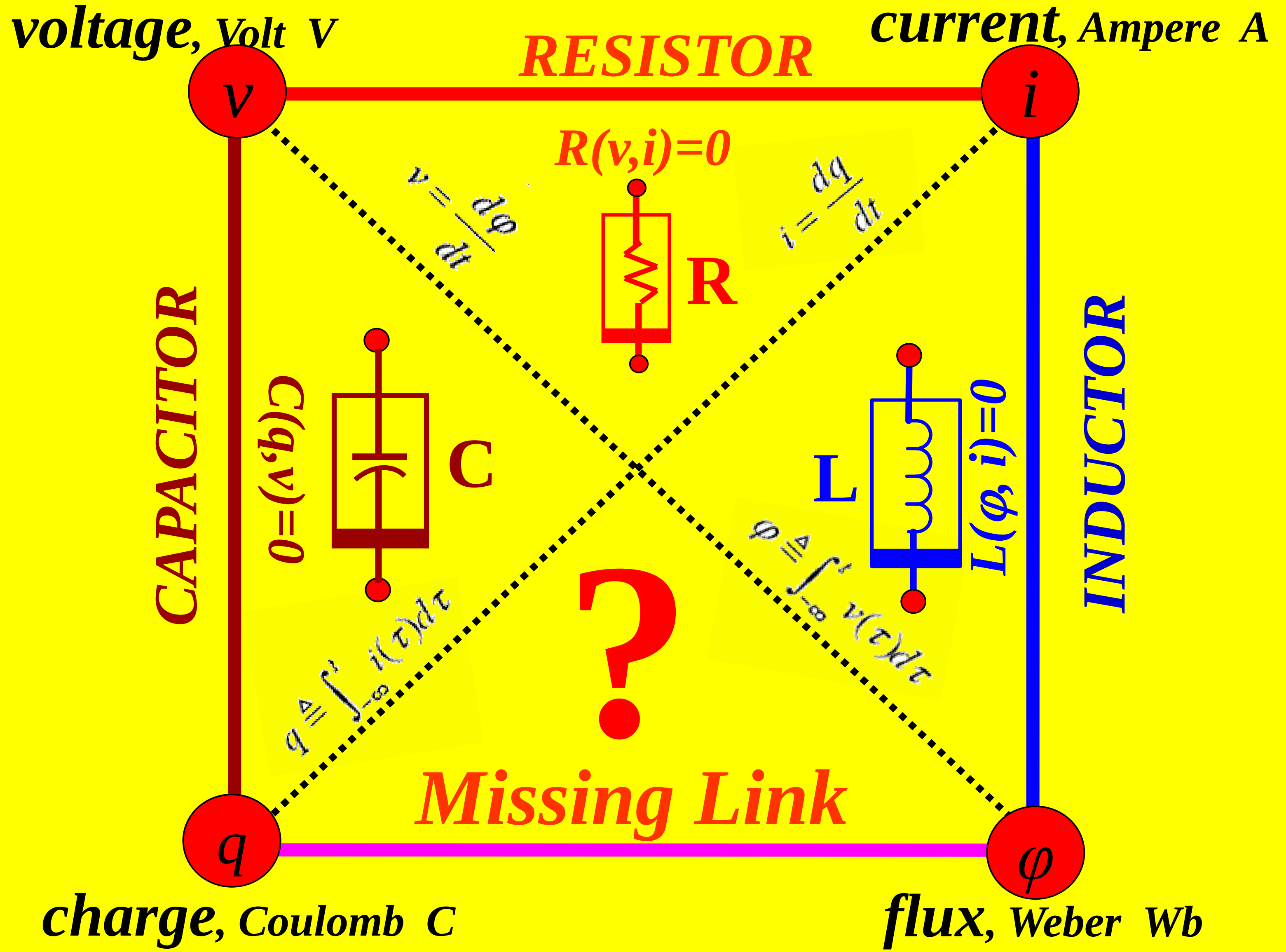
voltage, Volt V
current, Ampere A
RESISTOR
v
i
R(v,i)=0
R
q
φ
charge, Coulomb C
flux, Weber Wb



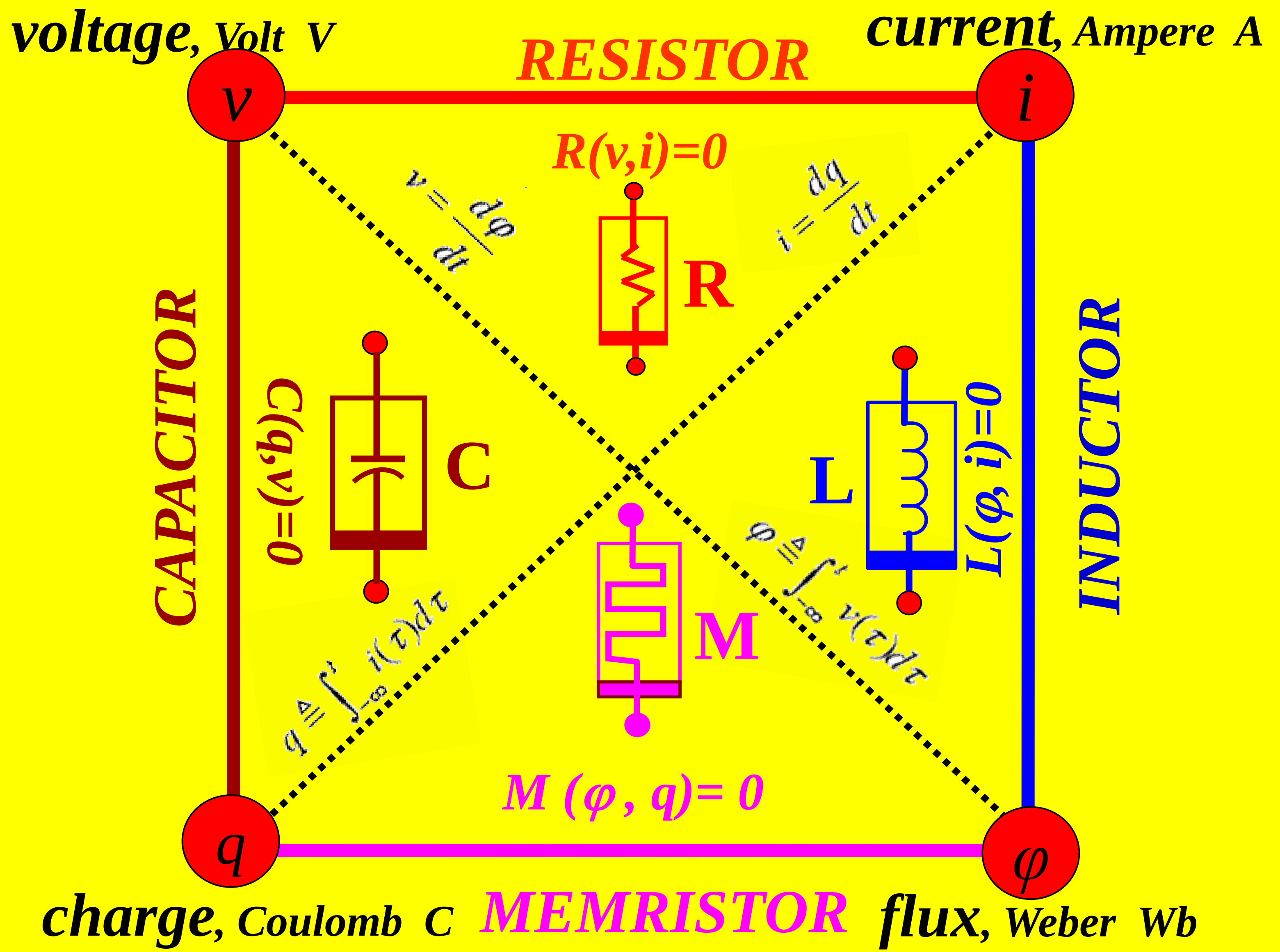
voltage, Volt V
current, Ampere A
RESISTOR
v
i
R(v,i)=0
$v = \frac{d\varphi}{dt}$
$i = \frac{dq}{dt}$
R
L
L(φ, i)=0
INDUCTOR
$\varphi \triangleq \int_{-\infty}^{t} v(\tau)d\tau$
$q \triangleq \int_{-\infty}^{t} i(\tau)d\tau$
q
φ
charge, Coulomb C
flux, Weber Wb


voltage, Volt V
current, Ampere A
RESISTOR
v
i
R(v,i)=0
$v = \frac{d\varphi}{dt}$
$i = \frac{dq}{dt}$
R
CAPACITOR
C(q,v)=0
C
L
L(φ, i)=0
INDUCTOR
$\varphi \triangleq \int_{-\infty}^{t} v(\tau)d\tau$
$q \triangleq \int_{-\infty}^{t} i(\tau)d\tau$
q
φ
charge, Coulomb C
flux, Weber Wb

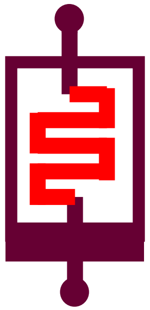
4 Basic Circuit Elements

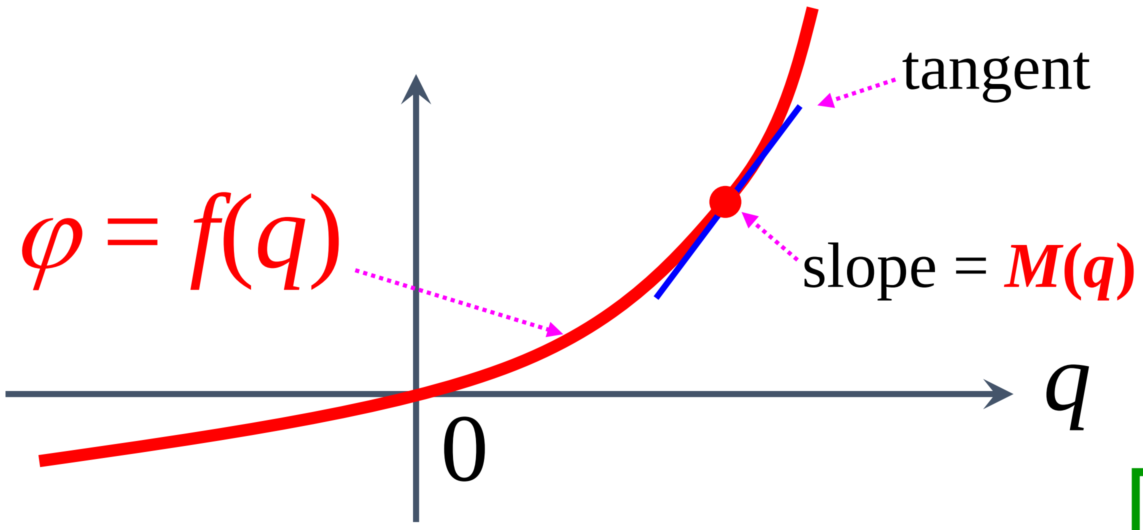


4 Basic Circuit Elements



Memristor





$$v = \frac{d\varphi}{dt} \equiv \underbrace{\frac{df(q)}{dq}}_{M(q)} \cdot \underbrace{\frac{dq}{dt}}_{i}$$

$$\varphi = f(q)$$

$$v = M(q)\ i$$

M(q) is called the ***Memristance.***

A Fourth Basic Element

Called the

Memristor

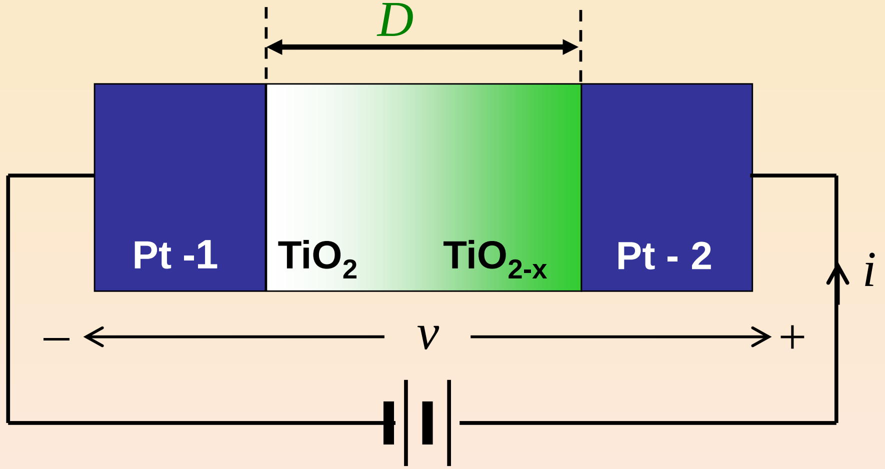
was postulated in **1971**

Leon O. Chua
Memristor : The missing circuit element
IEEE Transactions on Circuit Theory, vol.18, no.5, p.507-519, 1971.

and found in **2008**

D. B. Strukov, G. S. Snider, D. R. Stewart, and R. S. Williams
The Missing Memristor Found
Nature, vol.453, p.80-83,2008.

HP Memristor



$$v = M(q)\, i$$

Memristance

$$M(q) = R_{OFF}\left(1 - \frac{\mu_v R_{ON}}{D^2}\, q\right)$$

where

D is the device thickness (can be scaled to less than 2 nano meters)

R_{OFF}, R_{ON}, μ_v are device parameters

Memristor

is defined by a

State - Dependent

Ohm's Law

1973

Nobel

Prize

in

Physics

Discovers

Super-

conducting

Josephson

tunneling

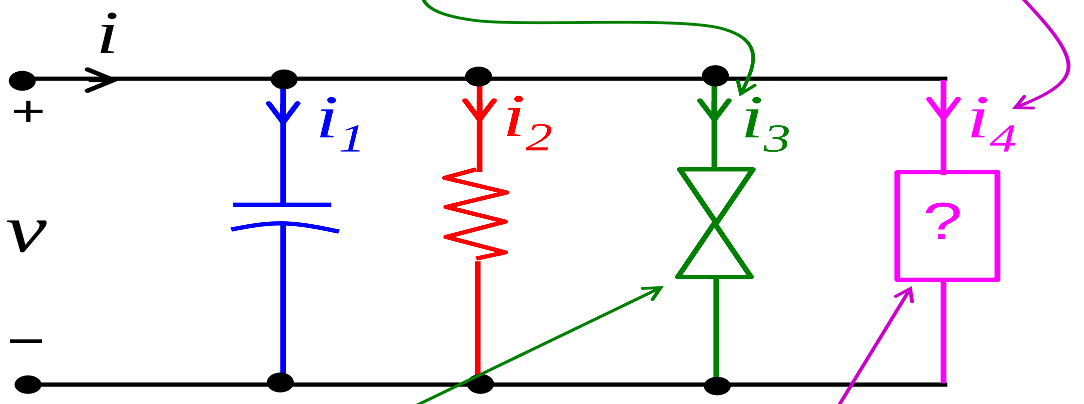
junctions

B. D. Josephson

Brian Josephson
1973 Nobel Prize in Physics:

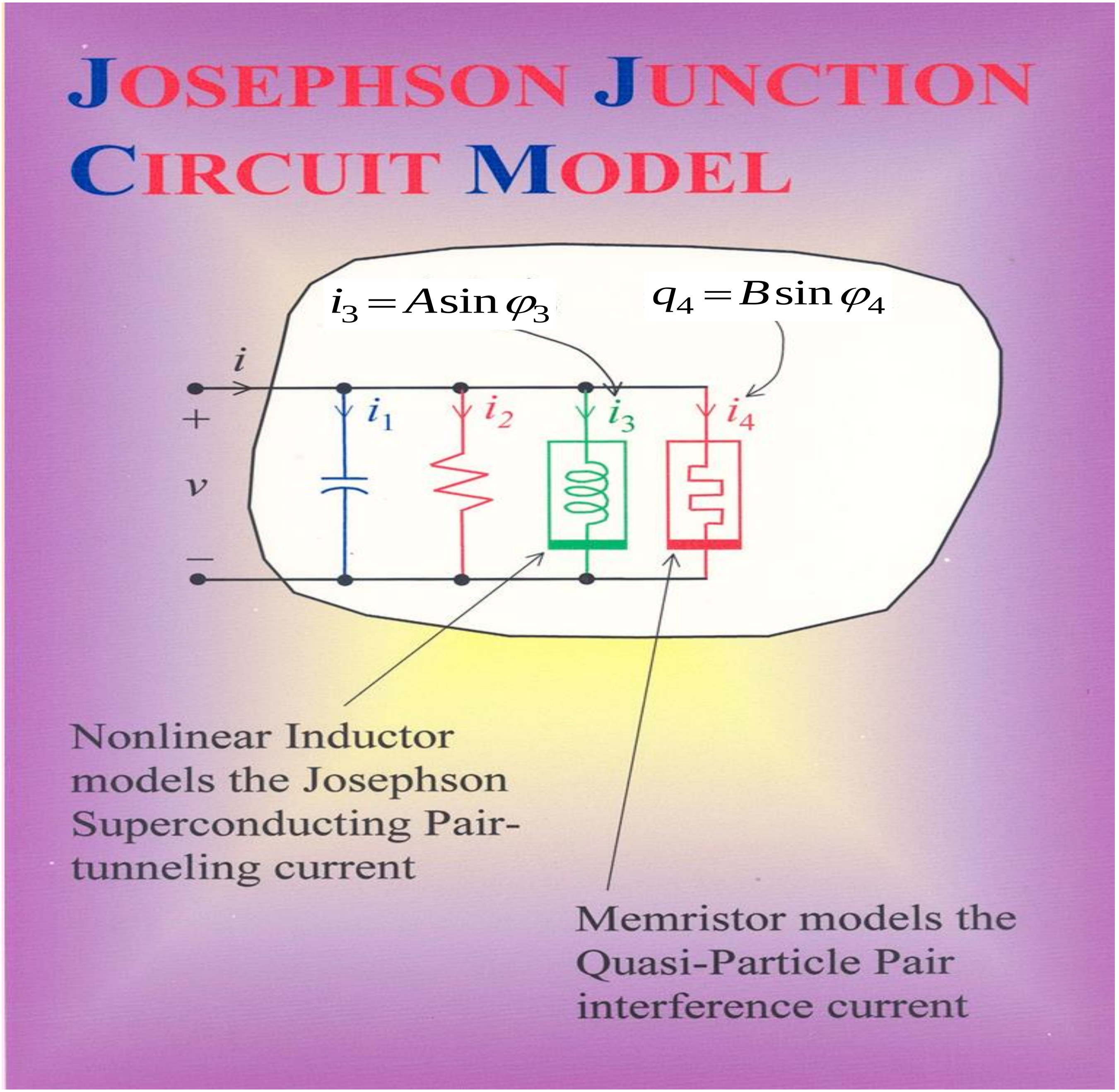
JOSEPHSON JUNCTION CIRCUIT MODEL

$i_3 = A \sin \varphi_3$ $i_4 = [B \cos \varphi_4] v_4$

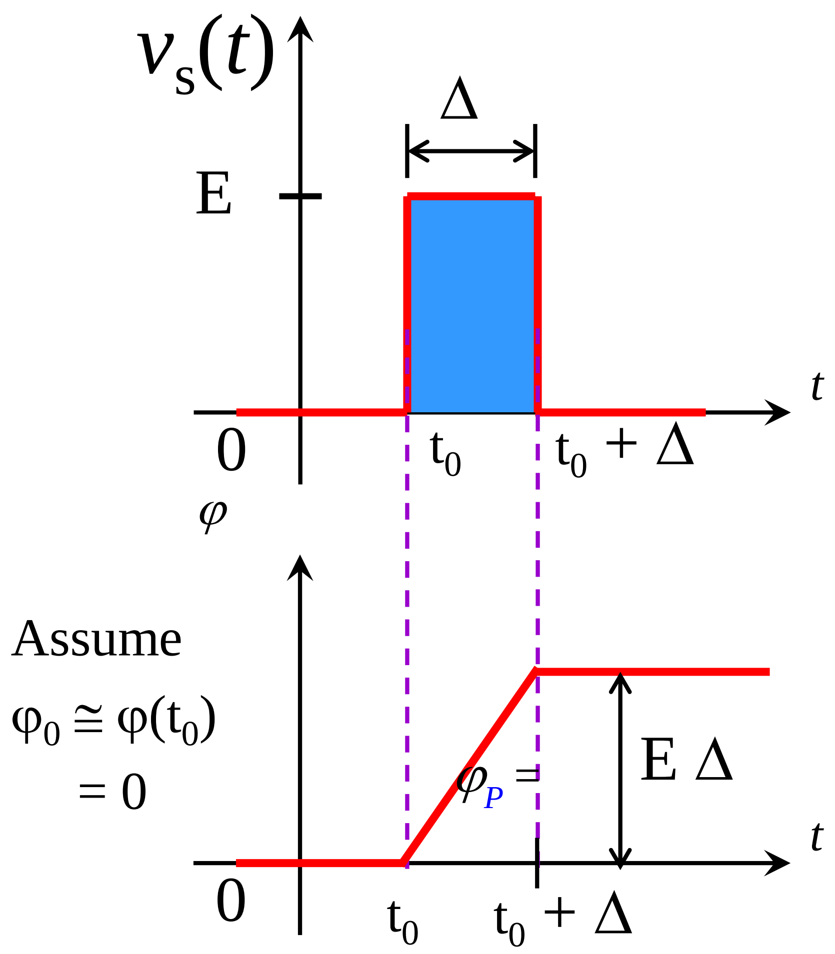


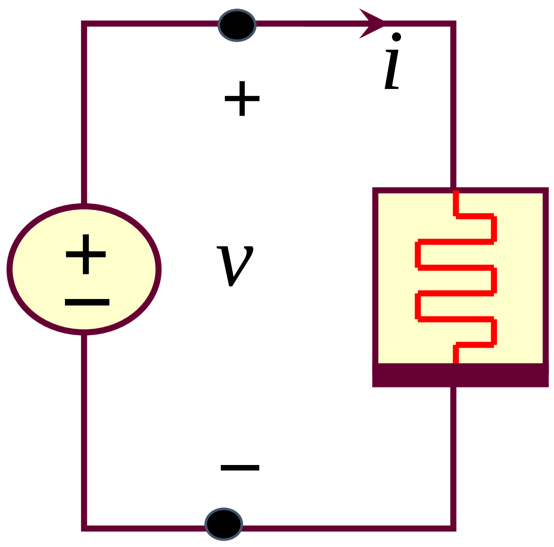
2-terminal element to model the Josephson Pair-tunneling current

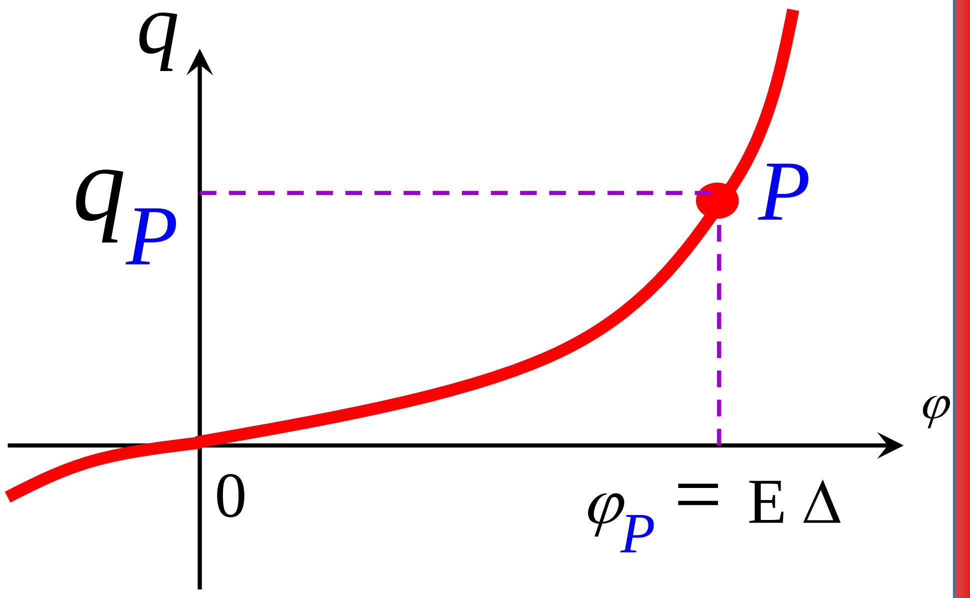
2-teminal element to model the Quasi-Particle Pair interference current


JOSEPHSON JUNCTION
CIRCUIT MODEL
$i_3 = A \sin \varphi_3$
$q_4 = B \sin \varphi_4$
i
+
v
−
i_1
i_2
i_3
i_4
Nonlinear Inductor models the Josephson Superconducting Pair-tunneling current
Memristor models the Quasi-Particle Pair interference current

Why is the Memristor Non-Volatile ?

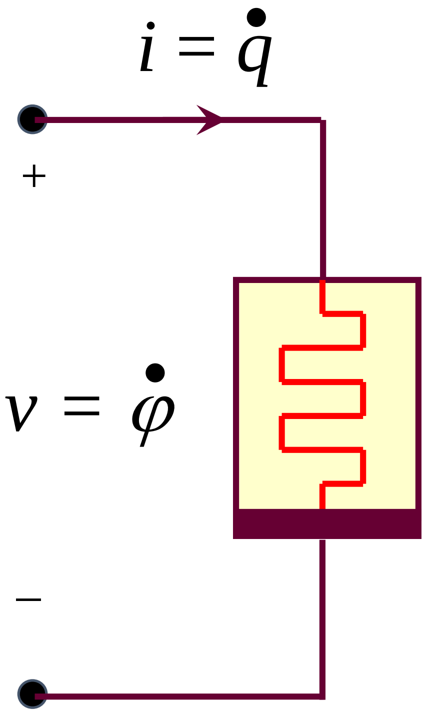






$$\varphi(t) = \varphi_0 + \int_{t_0}^{t} v(\tau)\, d\tau$$

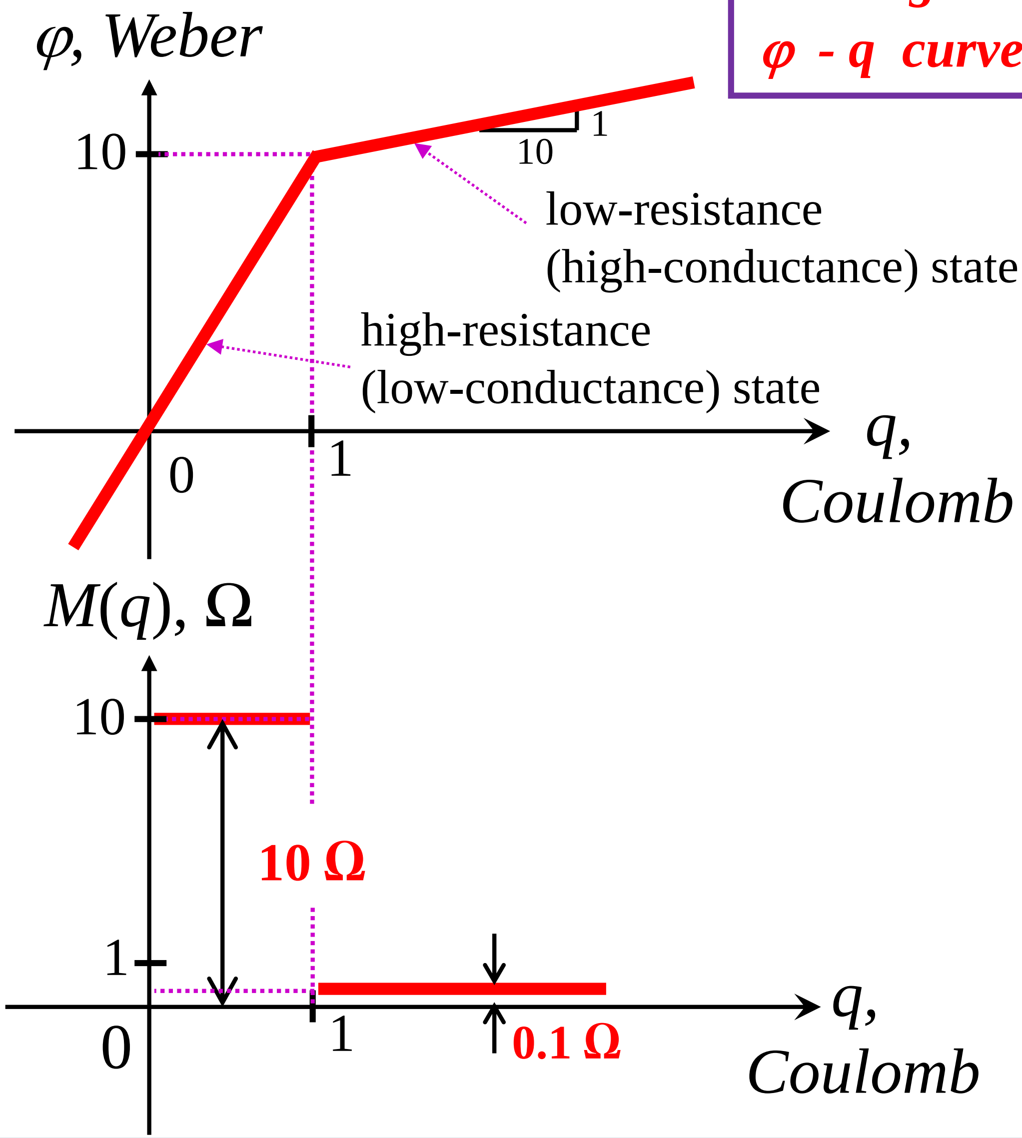
Example: A two-state Charge-Controlled Memristor



Charge-controlled memristor

$\varphi = \varphi(q)$

Charge - controlled φ - q curve : $\varphi = \varphi(q)$



Memristance

$$M(q) \triangleq \frac{d\varphi}{dq}$$

Non-volatile memories

are estimated to be a

400 billion dollar

Industry by 2020 !

Imagine a PC which turns on instantly !

Why not Flash ?

- *Can not* be economically scaled *below 10 nanometers*
- Poor Retention time: *Fails* after switching between *10,000 and 100,000 times*
- Low Speed
- Power Hungry
- They lose about *20 percent* of information for *decade*.

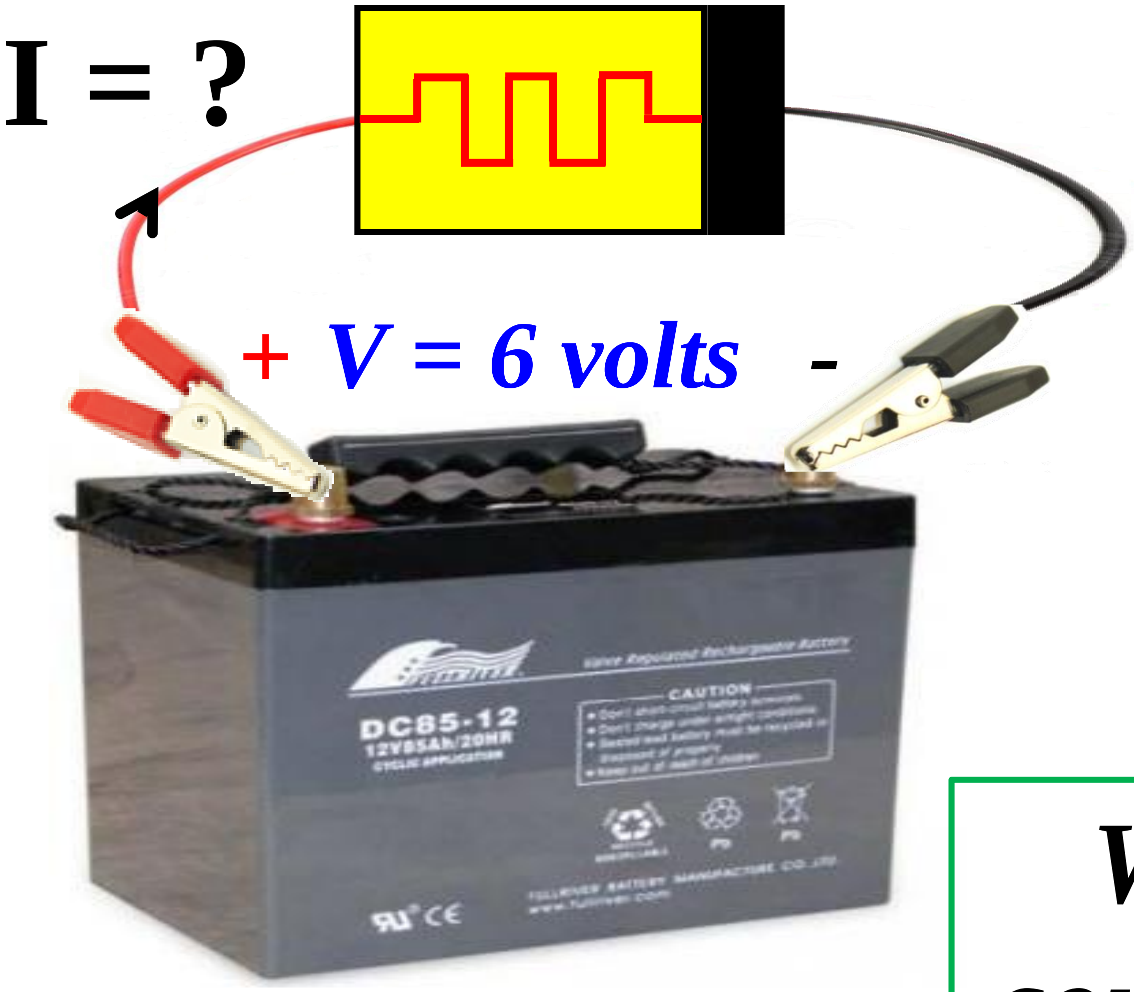
Non-Volatile Nano Memristors

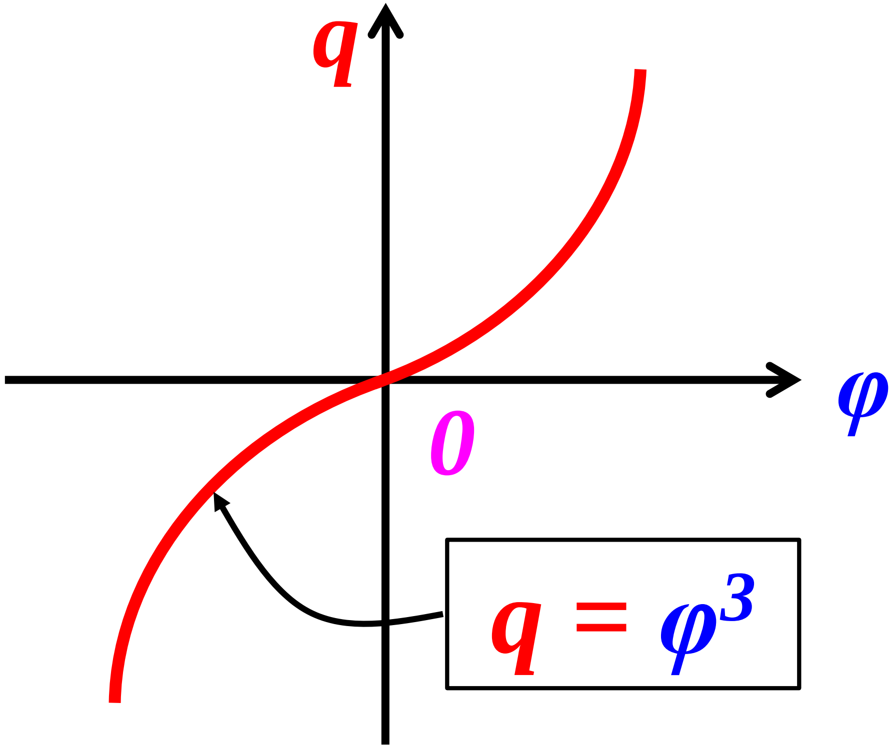
will eventually replace the following conventional computer memories

- ***Flash Memories***

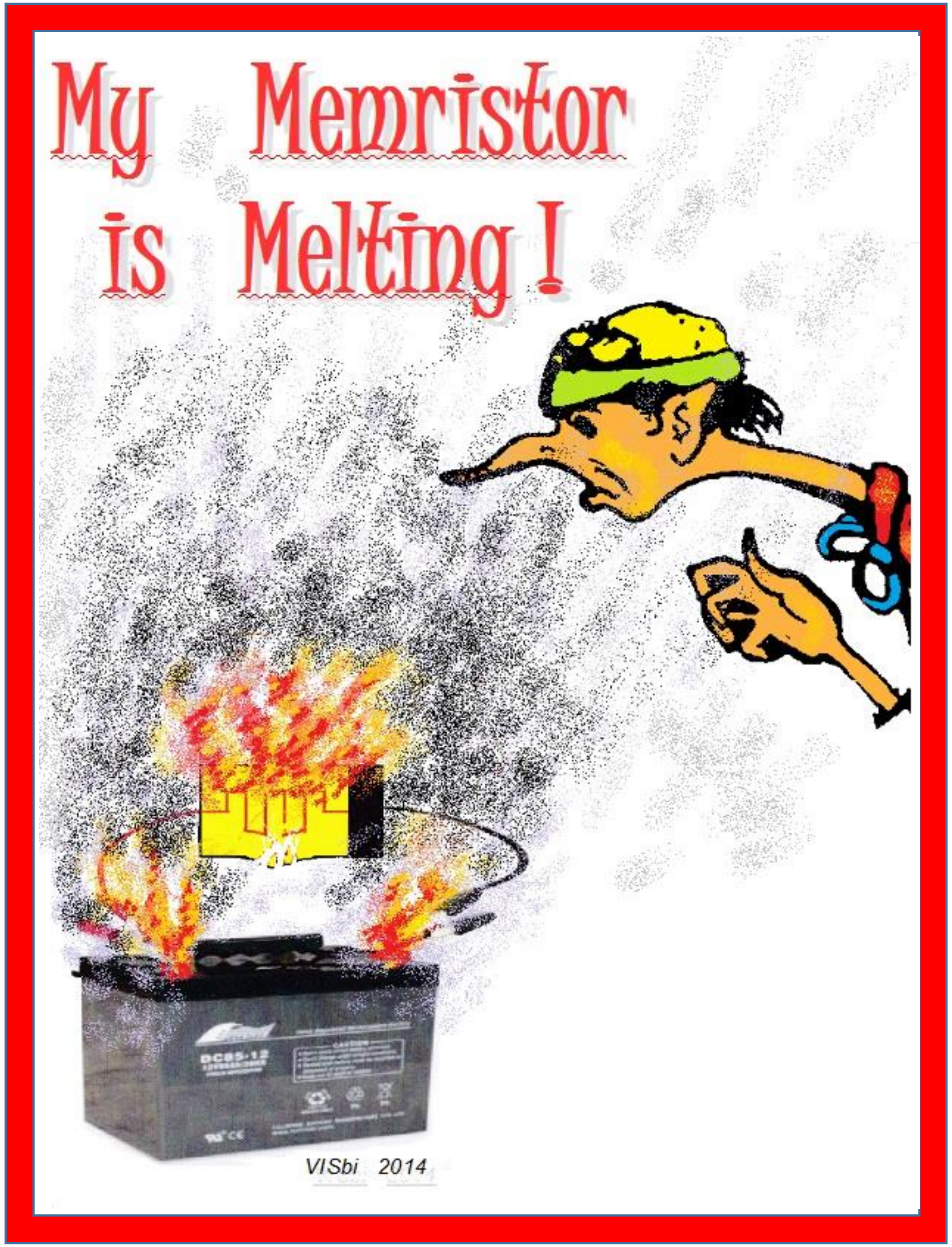
- ***DRAMs***

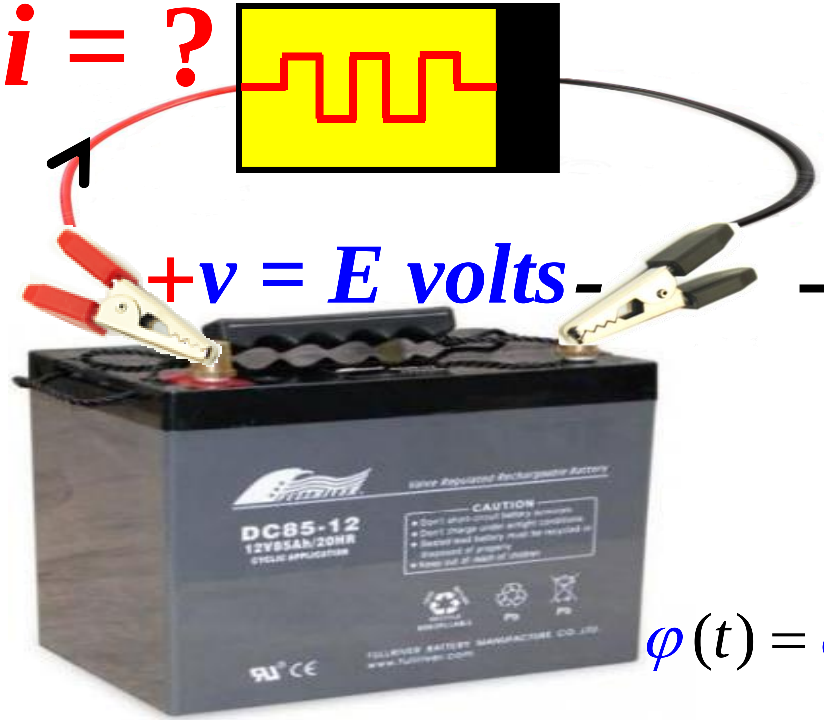
- ***Hard Drives***

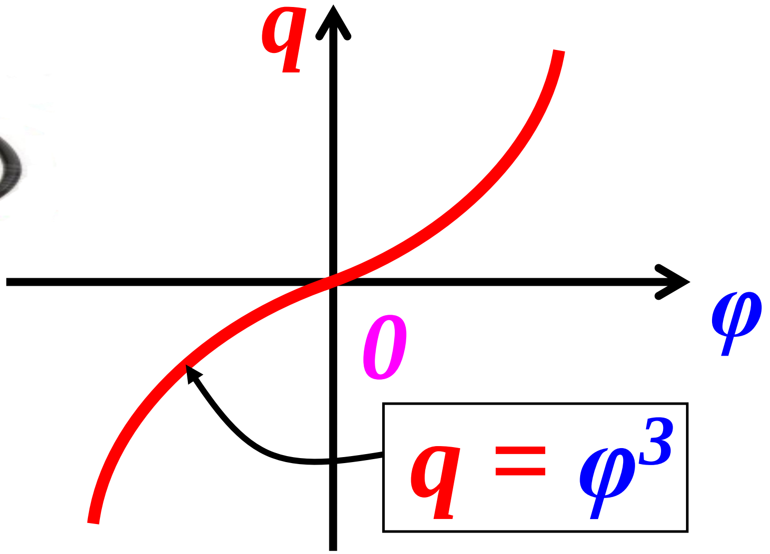


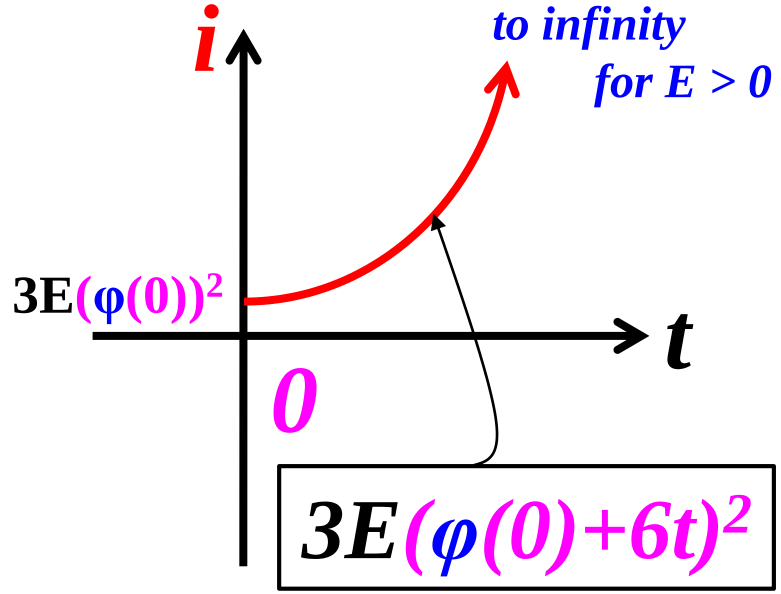


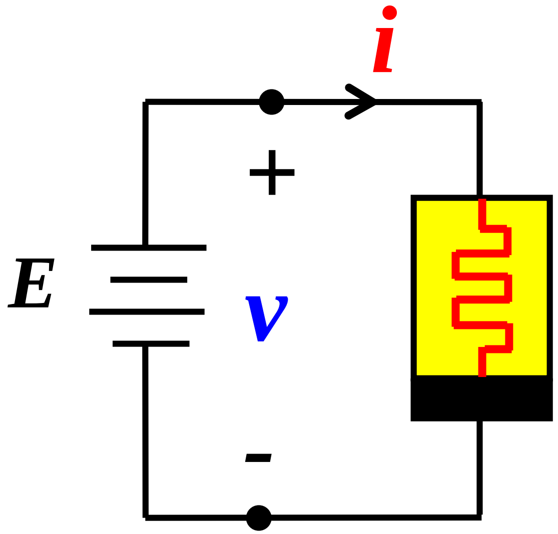
What happen when you connect a Memristor across a battery ?


My Memristor
is Melting !
VISbi 2014









$$\varphi(t) = \varphi(0) + \int_0^t v(\tau)\, d\tau$$

$$= \varphi(0) + E\,t$$

$$q(t) = \left(\varphi(0) + E\,t\right)^3$$

$$i(t) = \frac{dq(t)}{dt} = 3\left(\varphi(0) + E\,t\right)^2 (E)$$

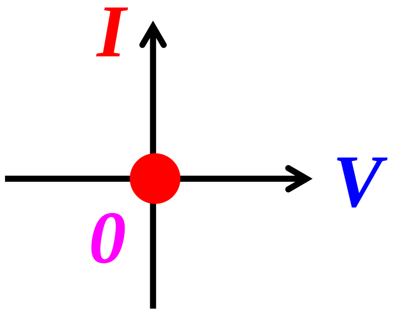
$$= 3E\left(\varphi(0) + E\,t\right)^2$$

$$\rightarrow \infty \quad as\ t \rightarrow \infty$$

Shocking Truth !

The DC V-I curve consists of only one point

(V, I) = (0, 0).



The

Ideal Memristor

does not have a

DC V-I Curve !

Standing Assumption

All *state variables* x_i in the *state equation*

$$\dot{x} = f(x, v)$$ (*Voltage-Controlled Memristor*)

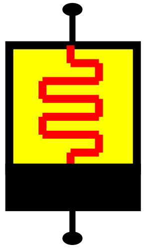
or

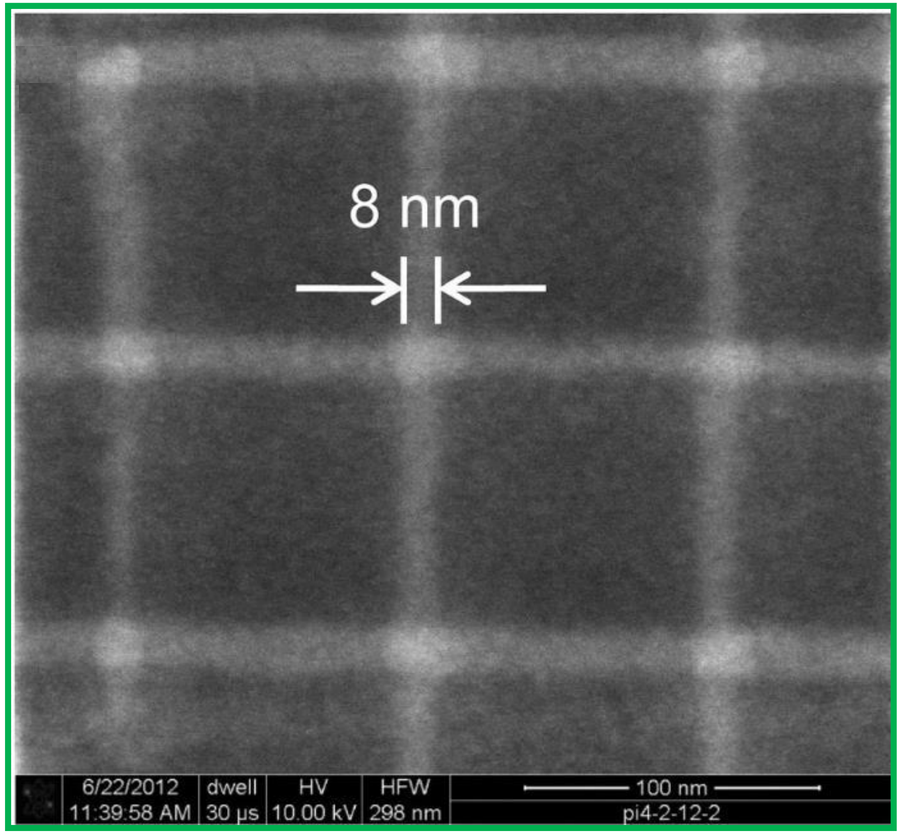
$$\dot{x} = f(x, i)$$ (*Current-Controlled Memristor*)

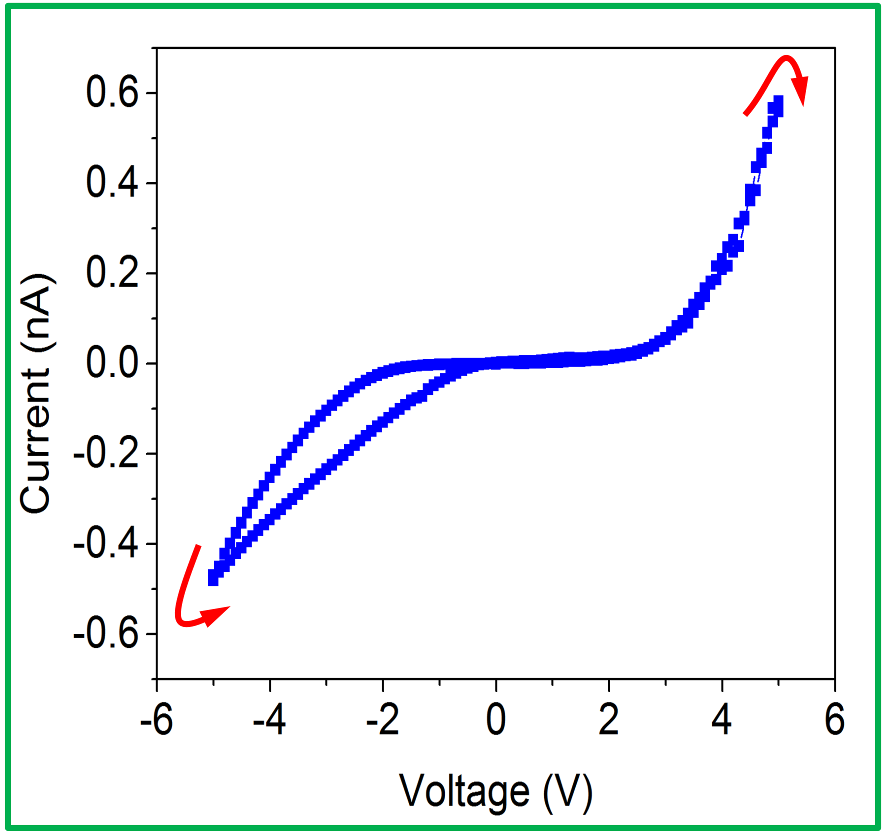
have *infinite* range:

$$-\infty < x_i < \infty$$

An 8 nm Memristor





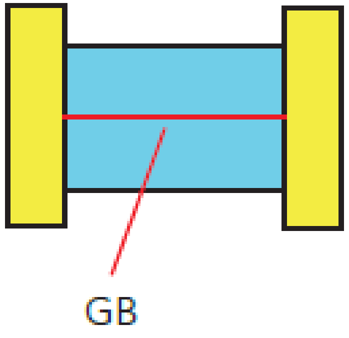


From:
S. Pi, P. Lin, Q. Xia, "Cross point arrays of 8 nm × 8 nm memristive devices fabricated with nano imprint lithography", J. Vac. Sci. Technol. B 31, 06FA02-1 - 06FA02-6, 2013

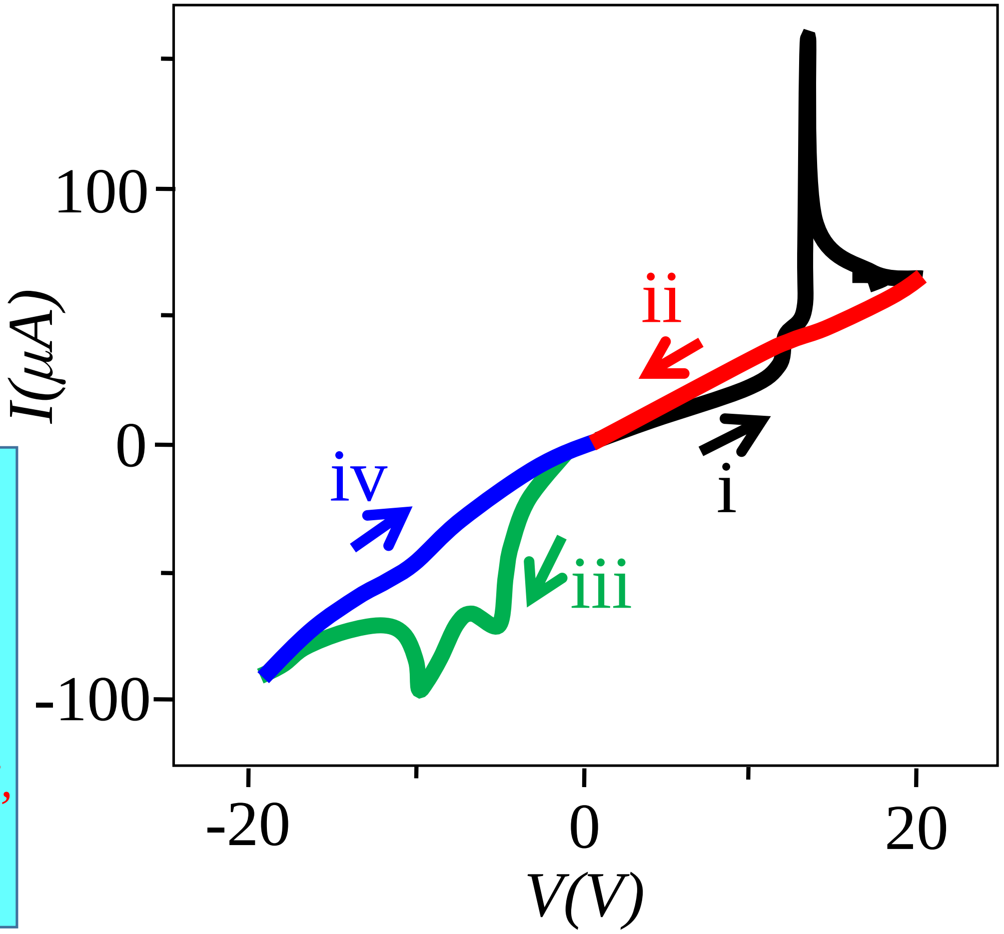
Memristor

made from

A single Layer of the Molecule MoS_2



From:
V. K. Sangwan, D. Jariwala, I. S. Kim, K. S. Chen, T. J. Marks, L. J. Lauhon, M. C. Hersam, "Gate-tunable memristive phenomena mediated by grain boundaries in single-layer MoS_2", Nature Nanotechnology 10, p. 403-406, 2015



How Do You Know Your *Device* is a *Memristor* ?

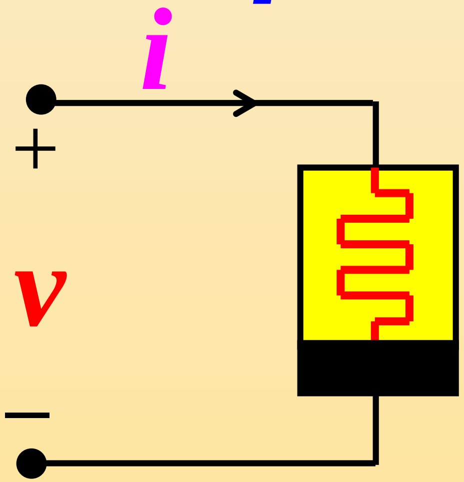
Since ***hp***'s 2008 publication in ***Nature*** of a *nano-scale memristor*, numerous other *memristors* have been published.

Less than ***5*** such publications have an *equation* describing their device ***!***

How then can they claim their device are memristors ?

Experimental Definition of the

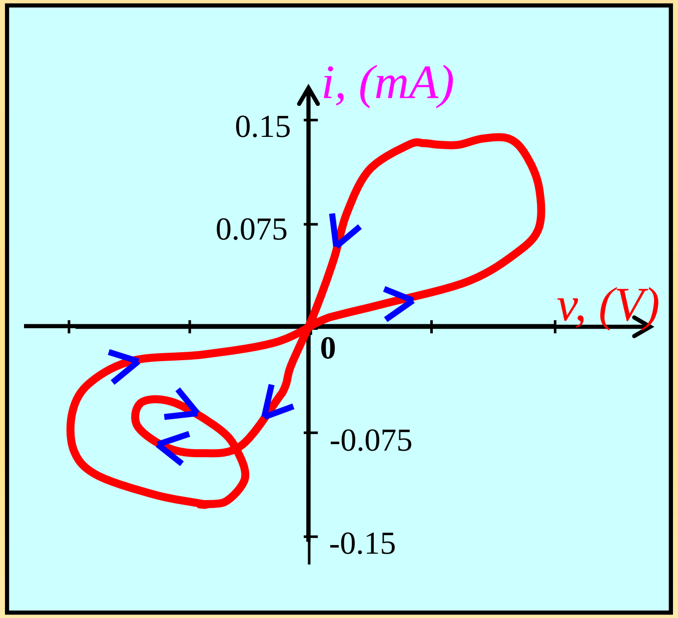
Memristor



If it's Pinched,

it's a

Memristor



Genealogy of Memristors

Extended Memristor

Voltage – Controlled

$$i = G(x, v)v$$

$$G(x, 0) \neq \infty$$

$$\frac{dx}{dt} = g(x, v)$$

→

Generic Memristor

Voltage – Controlled

$$i = G(x)v$$

$$\frac{dx}{dt} = g(x, v)$$

→

Ideal Generic Memristor

Voltage – Controlled

$$i = G(x)v$$

$$\frac{dx}{dt} = \hat{g}(x)v$$

→

Ideal Memristor

Voltage – Controlled

$$i = G(\varphi)v$$

$$\frac{d\varphi}{dt} = v$$

The Memristor Universe

EXTENDED MEMRISTOR

$$v = R(x, i)\, i \qquad R(x, 0) \neq \infty$$

$$\frac{dx}{dt} = f(x, i)$$

GENERIC MEMRISTOR

$$v = R(x)\, i$$

$$\frac{dx}{dt} = f(x, i)$$

IDEAL GENERIC MEMRISTOR

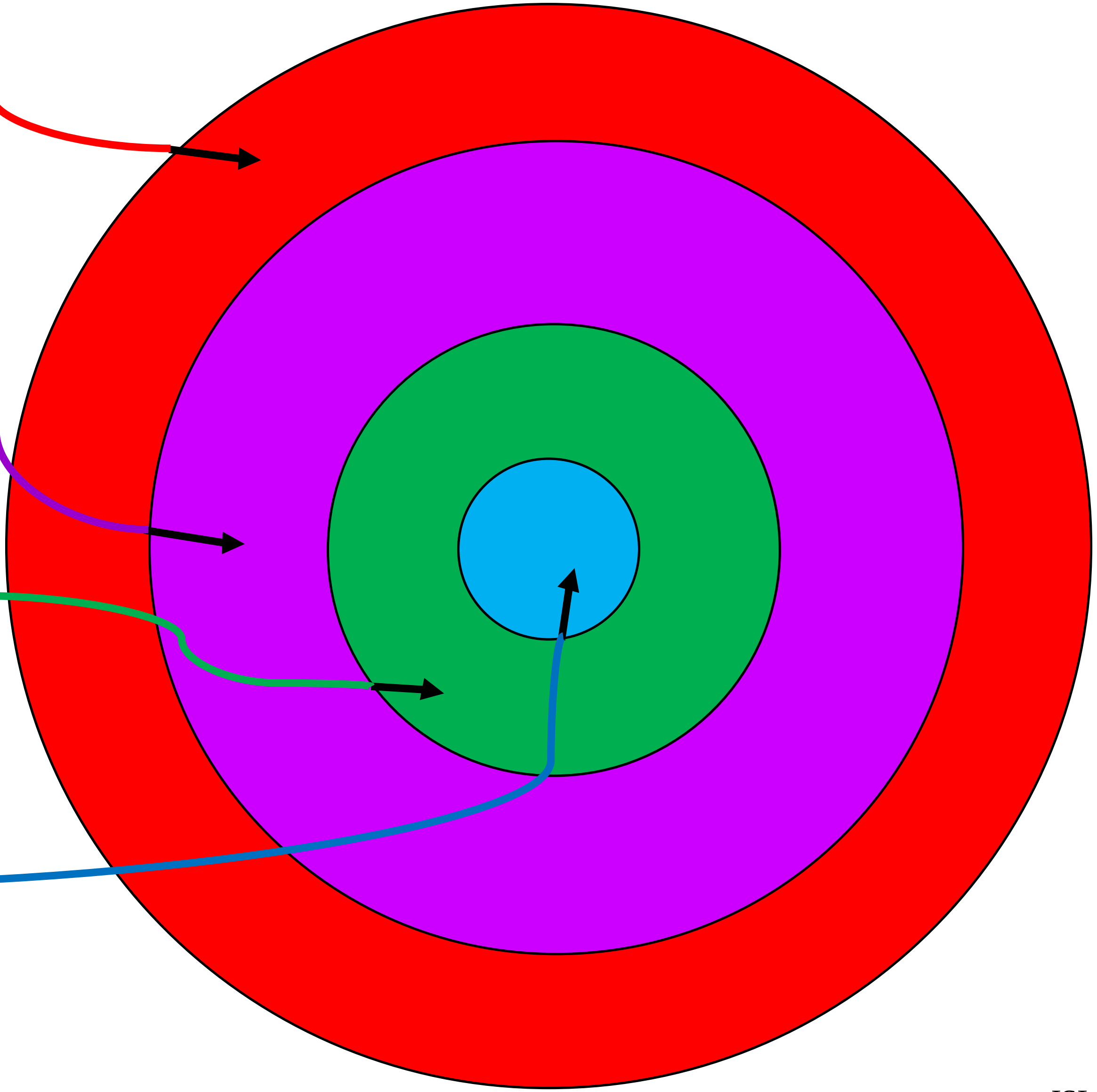
$$v = R(x)\, i$$

$$\frac{dx}{dt} = \hat{f}(x)\, i$$

IDEAL MEMRISTOR

$$v = R(q)\, i$$

$$\frac{dq}{dt} = i$$



Every

Ideal Memristor

spawns an

Infinite family

of

Equivalent

Generic Memristor

Siblings

Ideal Memristor Cousins

All *Generic* and *Extended* Memristors are ***Qualitatively Identical*** to Ideal memristors

All

Non-volatile Memories

based on

Resistance Switchings

are

Memristors

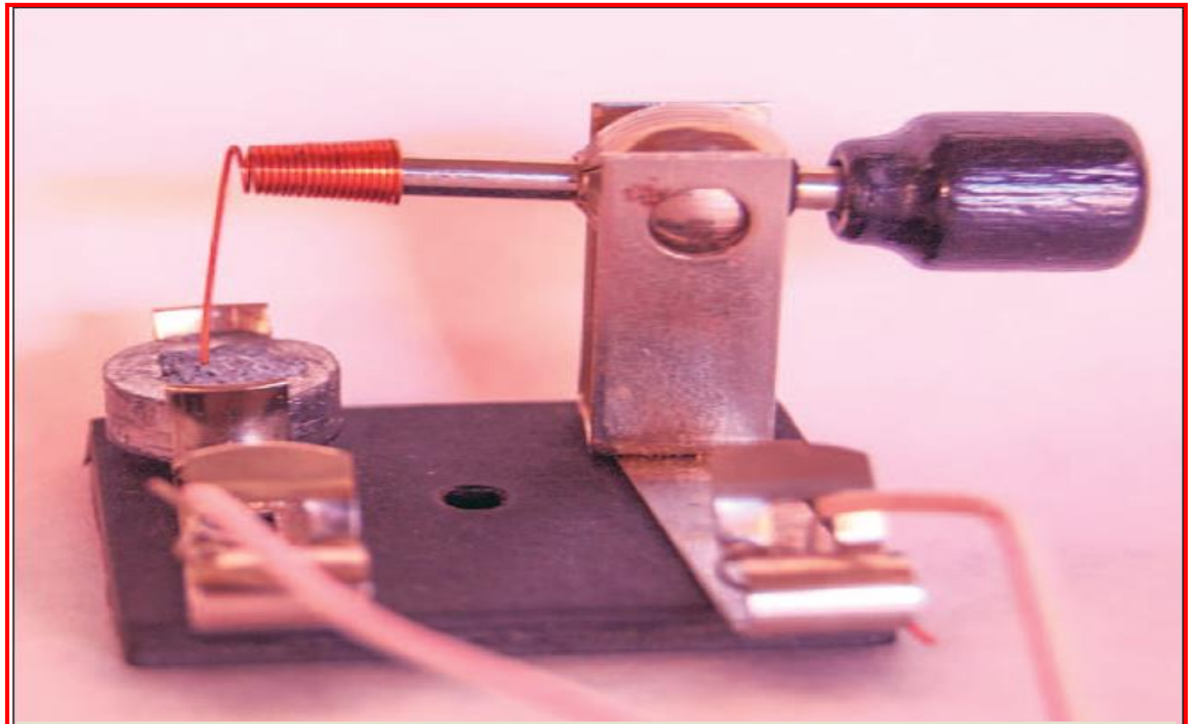
Following
non-volatile memory devices
are
memristors

- ***Re RAMS***
- ***Phase Change Memories***
- ***MRAMS***
- ***Ferro-Electric Non-volatile Memories***
- ***Atomic Switch***

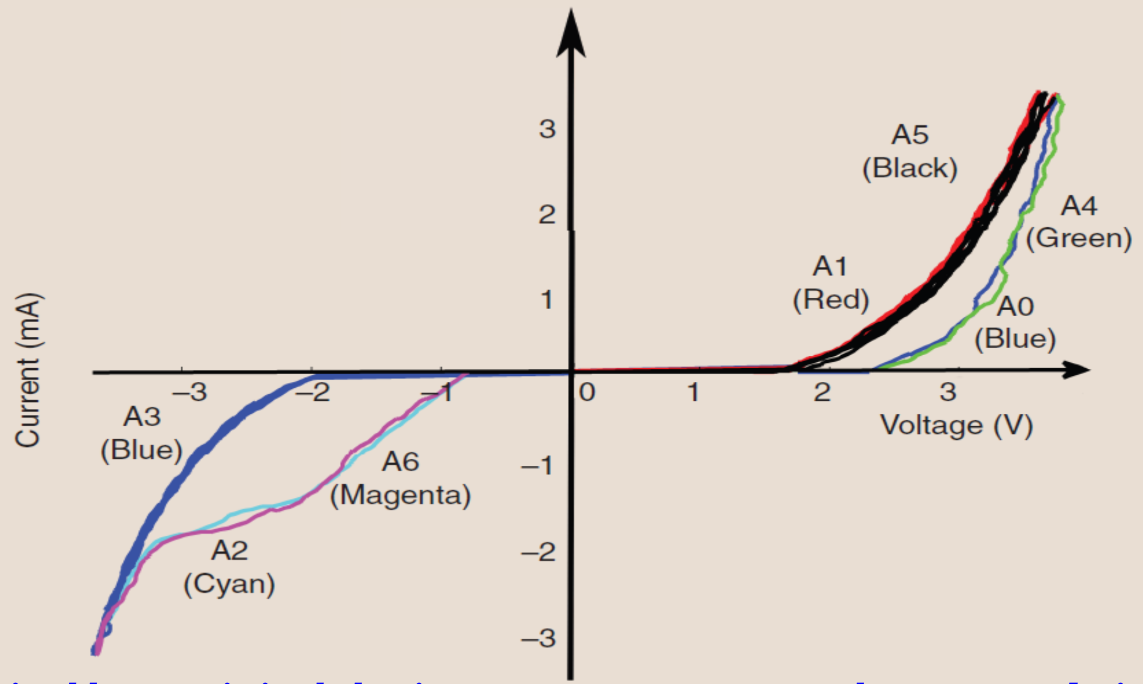
Examples of *Non-Volatile Memristors*

- **RRAM *Memristors*** (metal oxides Tio2, TaOx, etc.)
- **Polymeric *Memristors*** (conducting polymers)
- **Ferroelectric *Memristors*** (Ferroelectric films)
- **Manganite *Memristors*** (Perovskite manganite)
- **Spintronic *Memristors*** (spin-transfer torque magnetic layers)

Cat's Whisker from the First Radios are Memristors



Philmore cat's whisker in contact with a Galena crystal.



Bistable memristive behavior Input current versus voltage across device. One clearly observes pinched hysteresis loop for cat's whisker.

Material Views
www.MaterialViews.com

2012, 22, 4493-4499

Advanced Functional Materials
www.afm-journal.de

A Natural Silk Fibroin Protein-Based Transparent Bio-Memristor

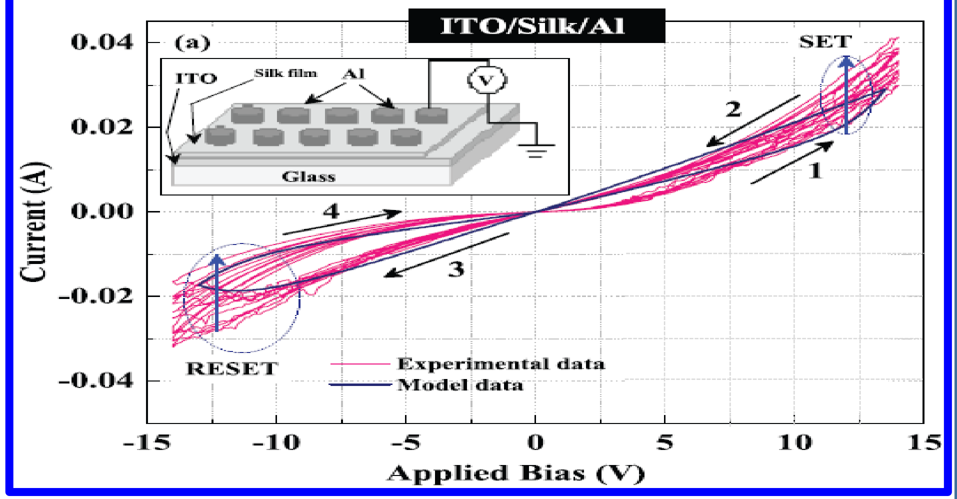
Mrinal K. Hota, Milan K. Bera, Banani Kundu, Subhas C. Kundu, and Chinmay K. Maiti

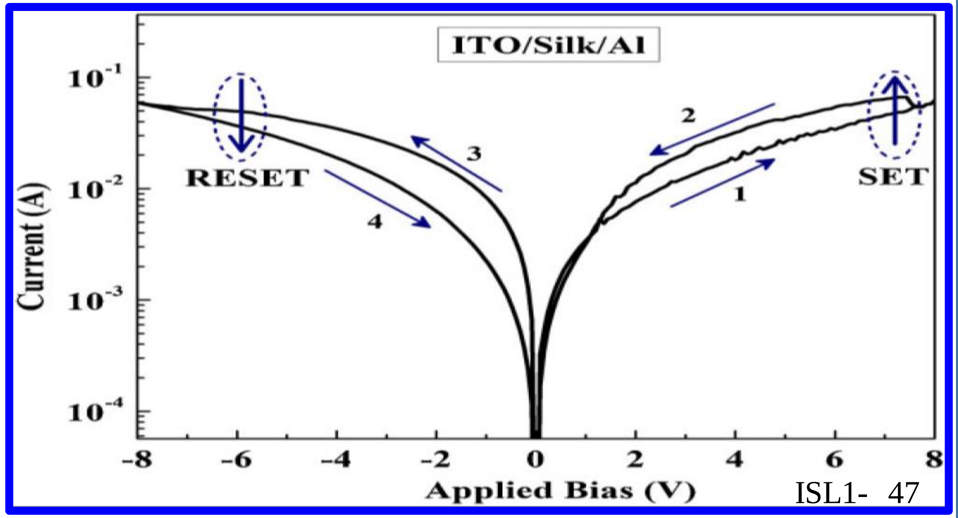

Cocoons of mulberry silkworm

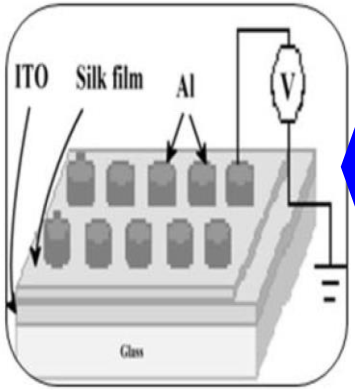

Cut pieces of cocoons


Degumming: removal of protein sericin


Degummed fibre of protein fibroin







Device Structure


Fibroin solution (conc. 2% w/v)


Thorough dialysis of fibroin solution to remove excess LiBr

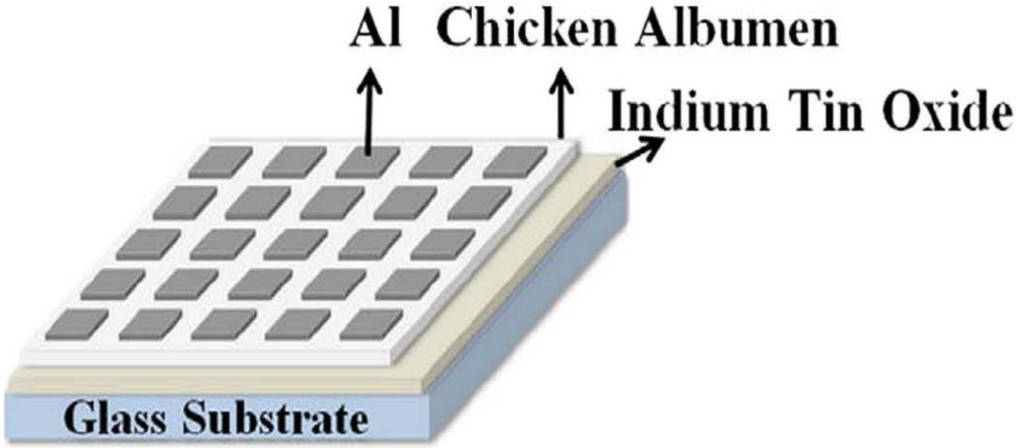

Dissolved fibre in 9.3 M LiBr

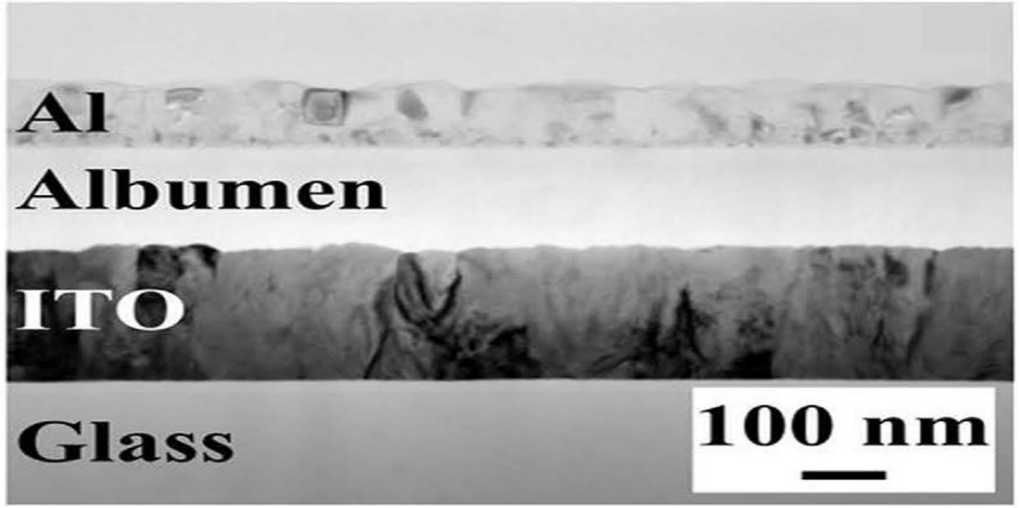
Pinched hysteresis loop in the *i – v* plane resembles a *seagull-wing* in the *log* | *i* | *- v* plane

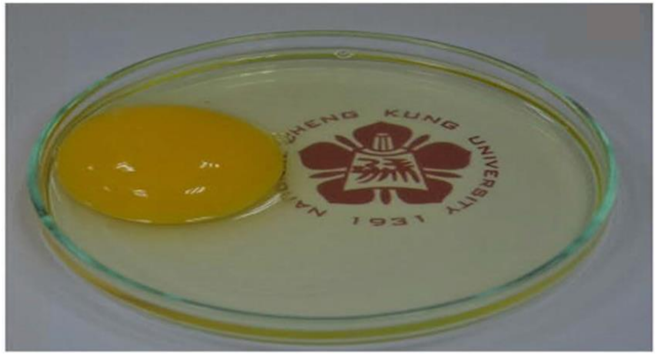
Scientific Reports 5 *Article Number: 10022* *www.nature.com/scientificreports*
Published: 7 May 2015

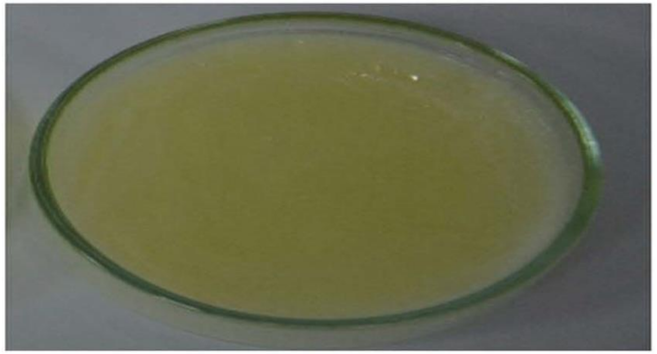
Nonvolatile Bio-Memristor Fabricated with Egg Albumen Film

Ying-Chih Chen, Hsin-Chieh Yu, Chun-Yuan Huang, Wen-Lin Chung, San-Lein Wu & Yan-Kuin Su

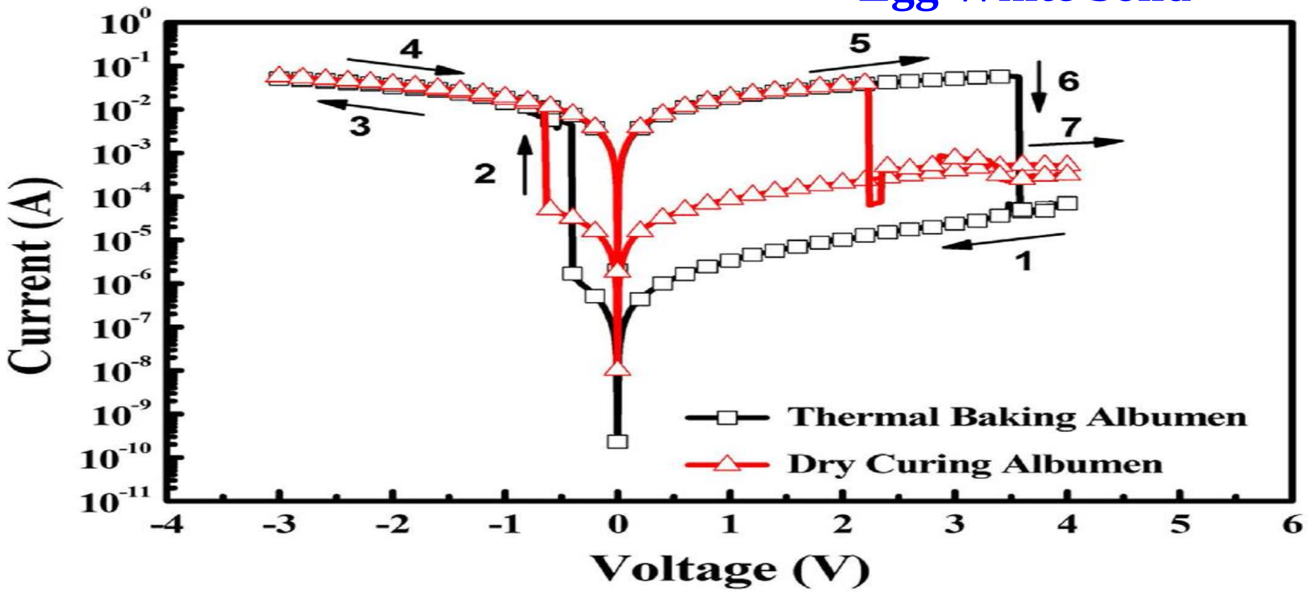








Egg White Solid



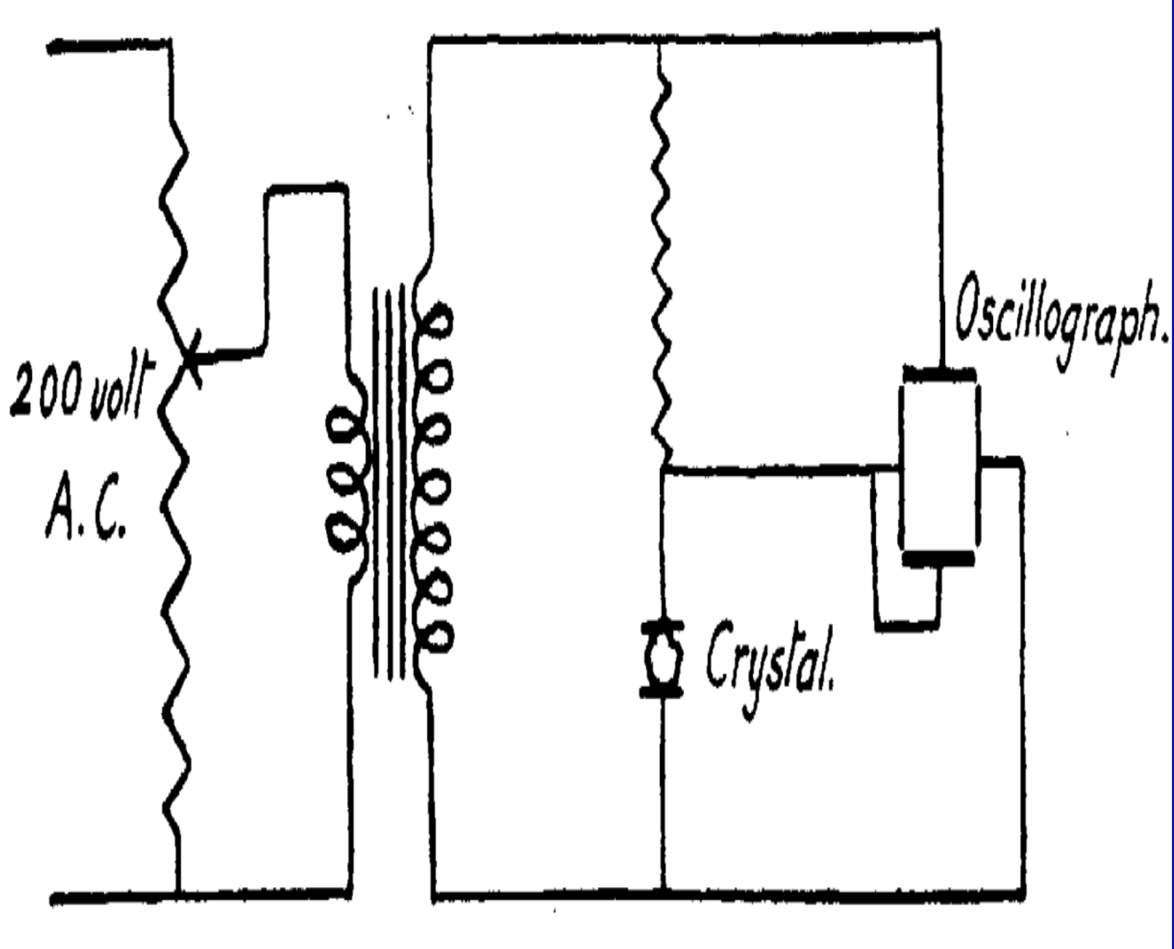
I–V characteristics of the thermal-baked and dry-cured albumen devices

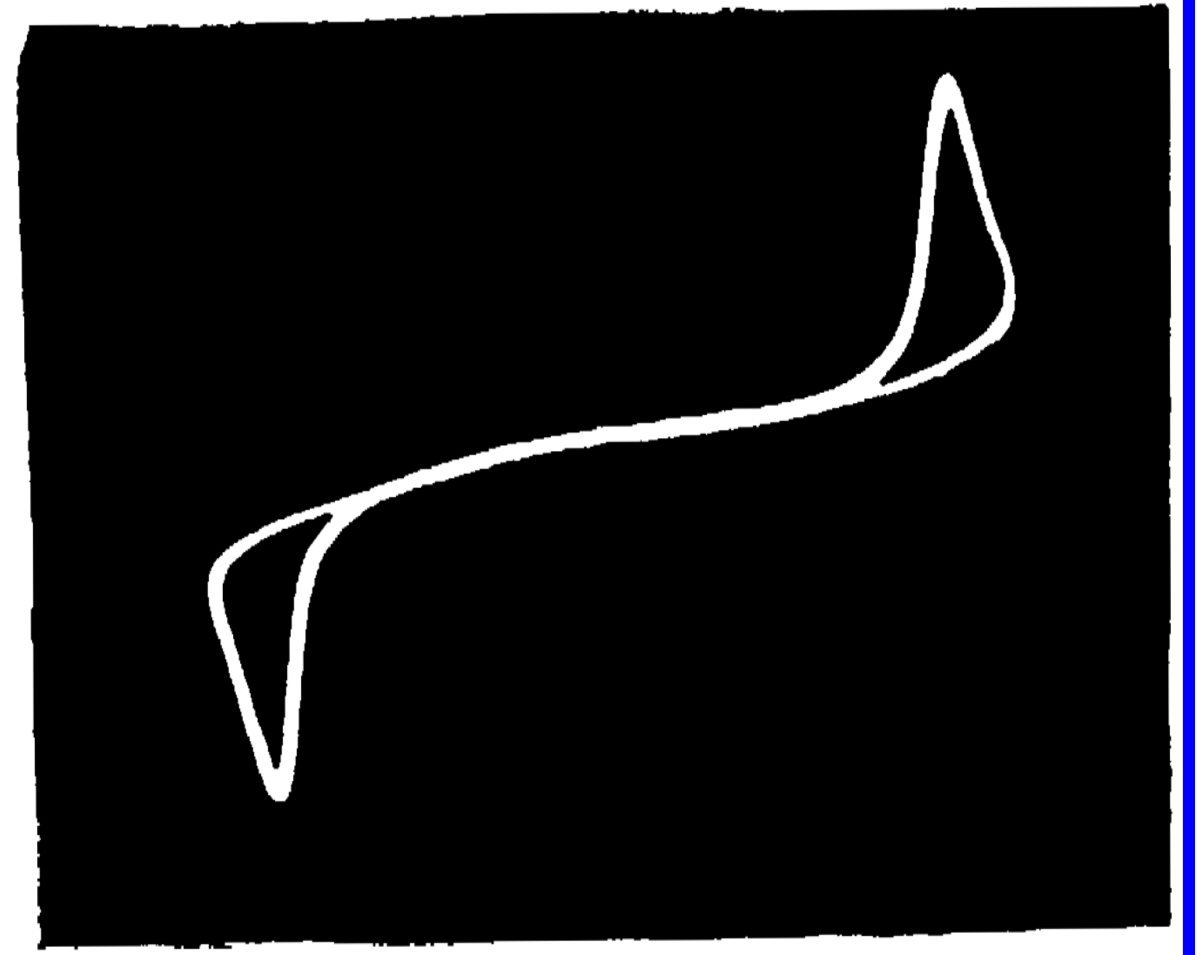
The *Quest* for building *nano-scale solid state non-volatile memories* dates back from *1939*

Trans. Faraday Society, Vol. 35, pp. 1436-1439, 1939

ELECTRICAL CONDUCTION OF COMMERCIAL BORON CRYSTALS

By J. H. BRUCE and A. HICKLING





ON THE PROPERTIES AND PREPARATION OF THE ELEMENT BORON.[1]

By E. Weintraub.

Received March 25, 1911.

Extracted from page 301

and the rapid change of resistance of boron with the temperature an accuracy in temperature measurements could be obtained which would be greater than anything yet available, especially as the boron resister could be introduced in form of a very small filament, thus disturbing but very little the thermal conditions. Of course the boron thermometer would have to be calibrated and above red heat it would have to be enclosed in an envelope filled with inert gas.

Closely connected with this would be the use of boron as a temperature regulator in a way so obvious as to require no particular description.

Finally, in the same line of thought, boron could be used for measuring radiant energy. A rough surface of boron would probably behave very nearly like a black body, but if necessary a part or the whole of its surface could be covered with fine carbon. One way in which the measurement of radiant energy could be carried out would be by determining the radiant energy input as a difference between electrical energy inputs before and after the radiant energy falls on the boron piece. The temperature of the boron piece is recognized to be the same by the fact that its resistance is the same. This ought to be a very delicate zero method.

The industrial applications, however, are those which have first claim on my attention. Without going into details, I may say that these are based on the electrical characteristics of boron and on its mechanical properties.

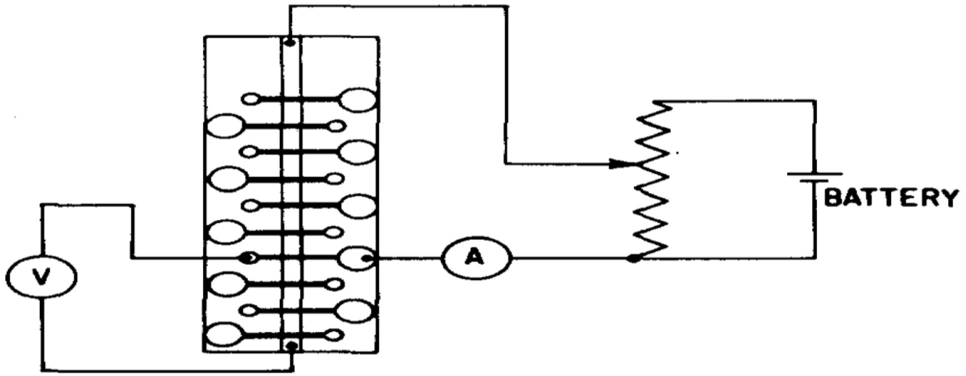
The large drop of resistance with the temperature which transforms boron under certain conditions from a very poor conductor for normal voltages into a good conductor for abnormally high voltages is certain to make it valuable for protection of electrical circuits.

The large drop of resistance with the temperature which transforms boron under certain conditions from a very poor conductor for normal voltages into a good conductor for abnormally high voltages is certain to make it valuable for protection of electrical circuits.

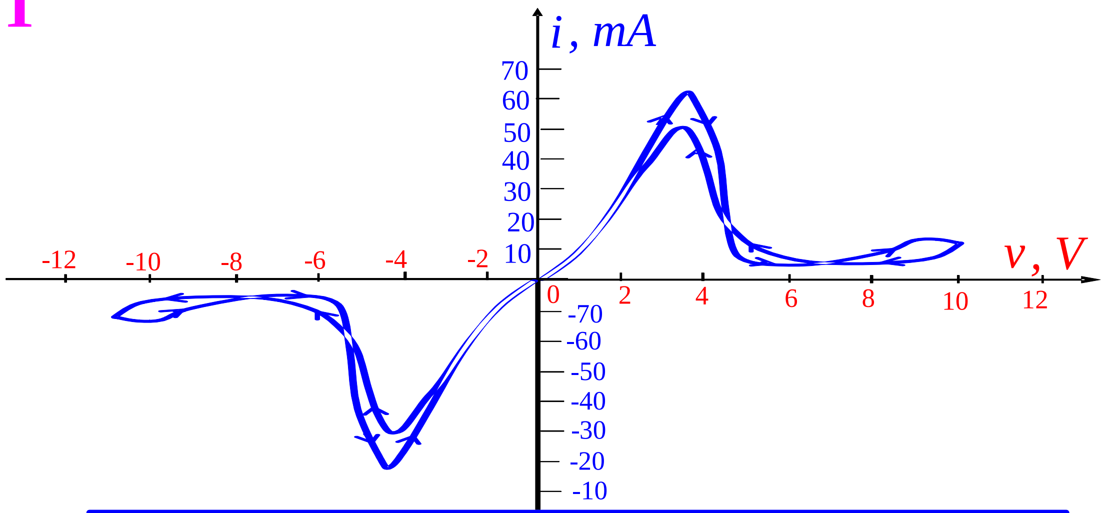
JOURNAL OF APPLIED PHYSICS *VOLUME 33, NUMBER 9* *SEPTEMBER 1962*

Low-Frequency Negative Resistance in Thin Anodic Oxide Films

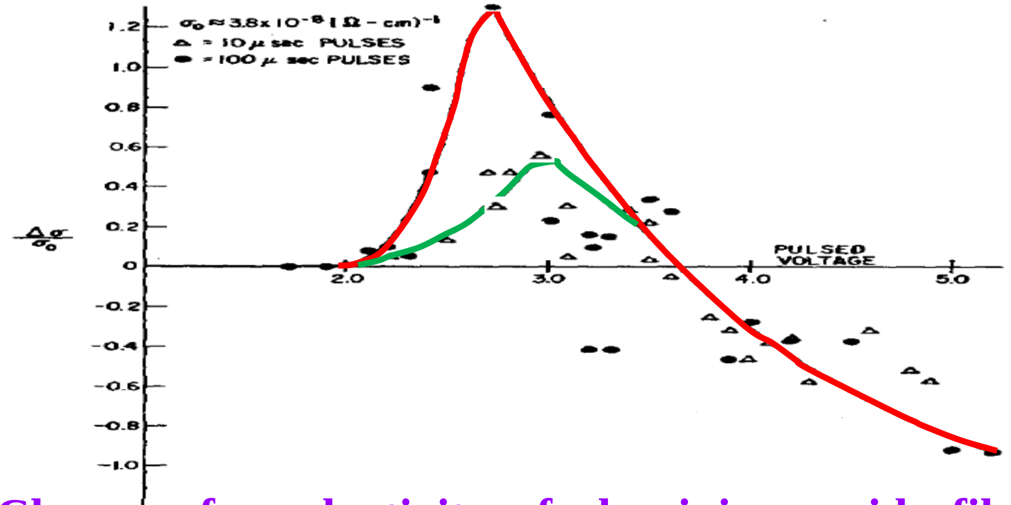
T. W. HICKMOTT



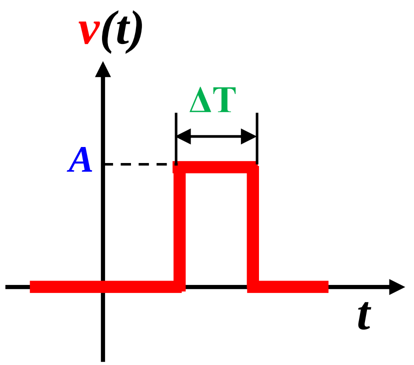
Preparation of metal-anodic oxide-metal sandwiches.
Circuit for measuring electrical characteristics.



Negative resistance is not found at 60 Hz !



Change of conductivity of aluminium oxide film by 10-*μsec* and 100-*μsec* pulses of varying voltage.



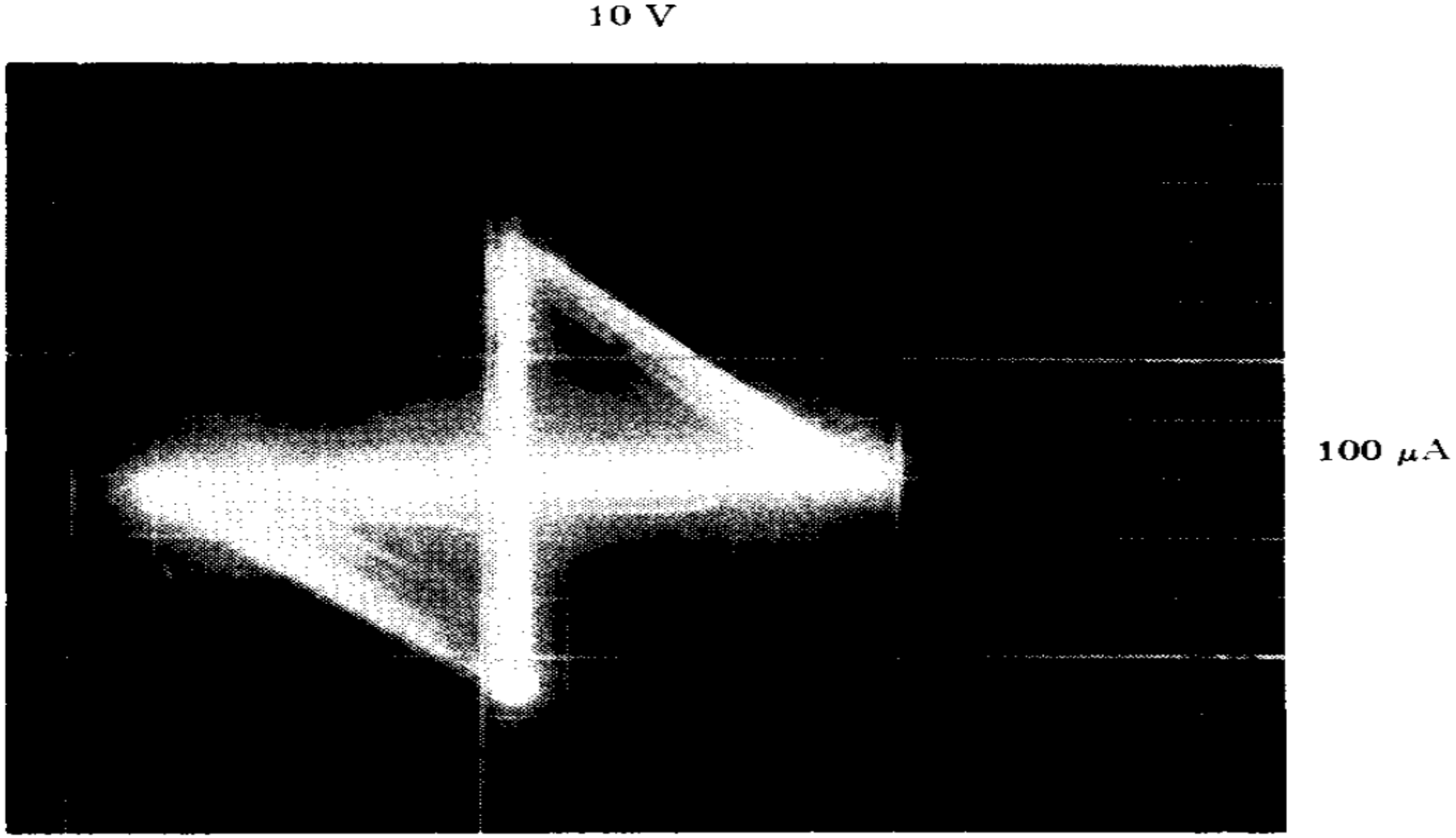
Conductance tuning voltage pulse

Small-signal *conductance* at zero DC bias voltage can be varied continuously over a wide range by applying *voltage pulses*, and *tuning* the *pulse amplitude A*, or the *pulse width* **ΔT** .

Journal of Applied Physics, Vol. 34, pp. 711-712, 1963

Negative Resistance in Thin Niobium Oxide Films

By S. PAKSWER and K. PRATINIDHI

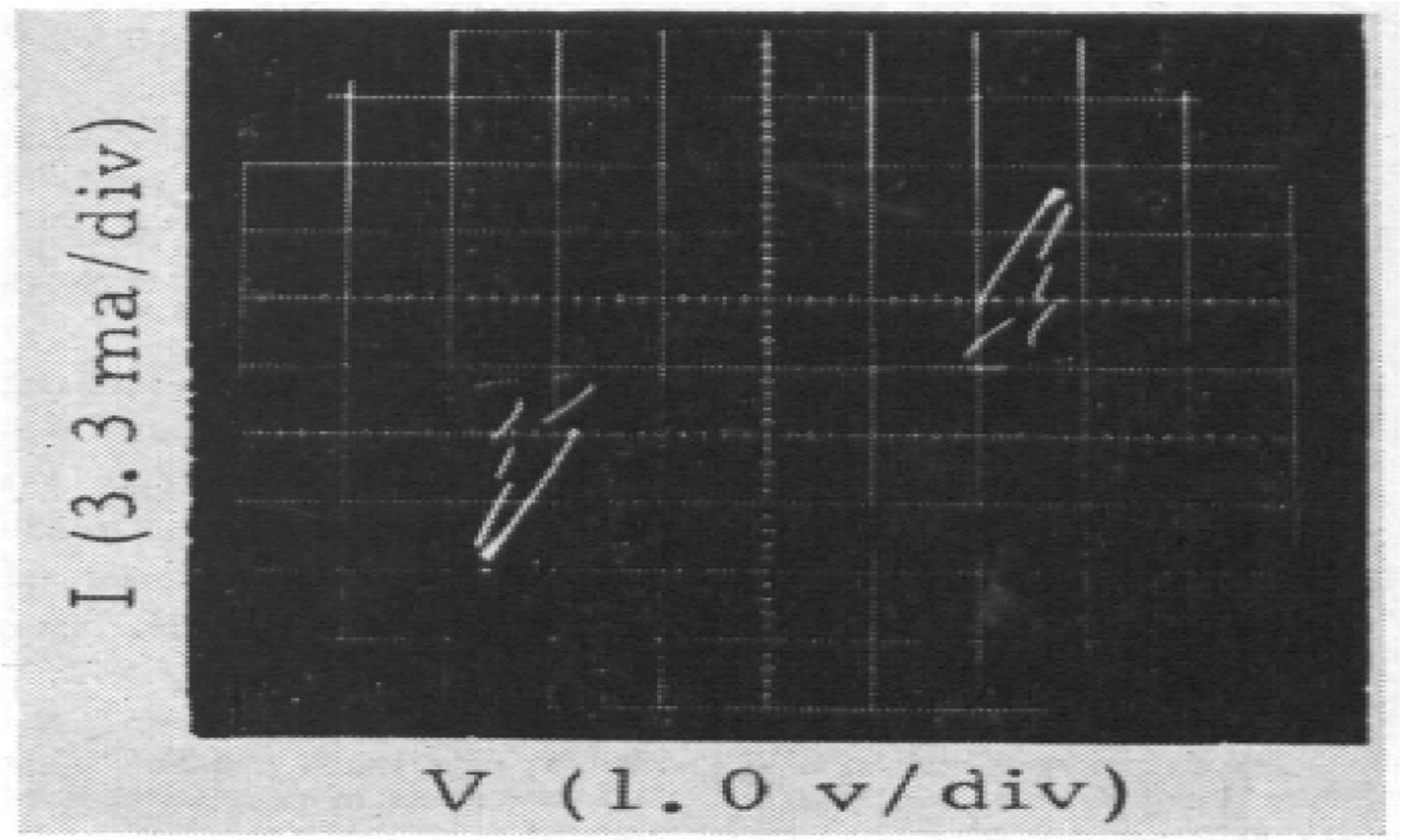


***I-V* characteristics of Al-Al_2O_3-Hg sandwich**

Proc. of the IEEE, Vol. 51, pp. 941-942, 1963

Current-Controlled Negative Resistance in Thin Niobium Oxide Film

By K. L. CHOPRA



***I-V* characteristics with multiple negative resistance regions**

Some *Excerpts of Confused* and *Ambiguous* statements on *Non-Volatile Memories*

- A "*memory*" *effect* where no *negative resistance* would normally occur….. [Hickmott, 1962; page 2673]
- *In all cases* when a new *memory state* is induced, *hysteresis* is manifest in the *V-I* characteristic and the *V-I* loop is generated….
- Furthermore, a *memory* state *never* accompanies a *V-I characteristic* that does not exhibit *hysteresis*…
- *Hysteresis* is also exhibited in the *V-I characteristic* when the *memory* is erased..
- In *no circumstances* is erasure observed when there is no *hysteresis*… [Simmons and Verderber, 1969; page 91]

First Hint of Pinched Hysteresis Loop:

A Device Dubbed

LETTER 8 MEMORY

was published in 1971

POLARIZED (LETETR '8') MEMORY IN CdSe POINT CONTACT DIODES

M. Kikuchi, M. Saito, H. Okushi

Electrotechnical Laboratory, Mukodai, Tanashi, Tokyo, Japan.

and

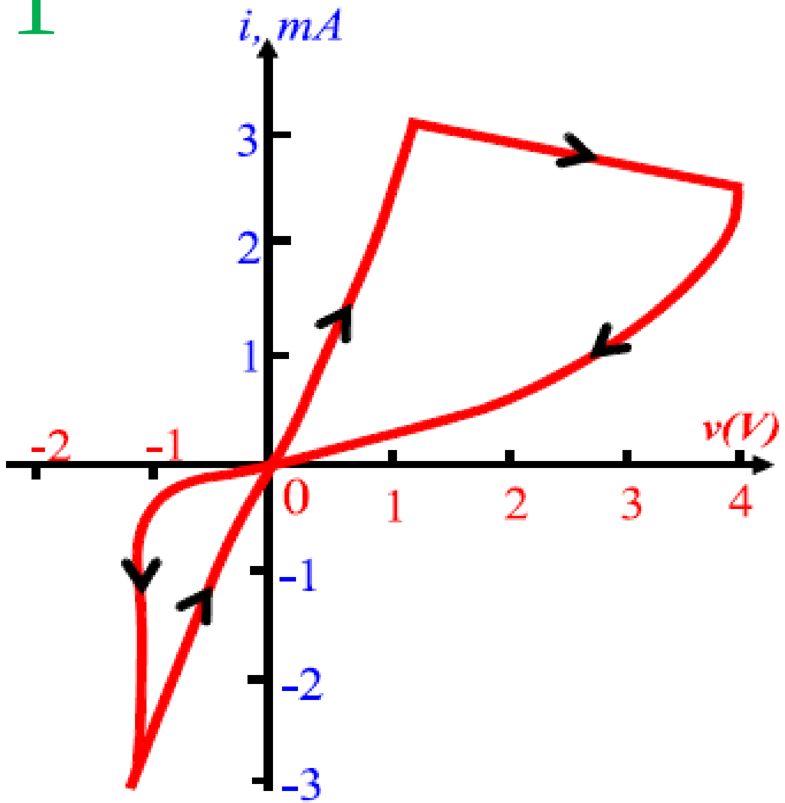
A Matsuda

Nippon Columbia Co. Ltd., Kawasaki, Kanagawa, Japan.

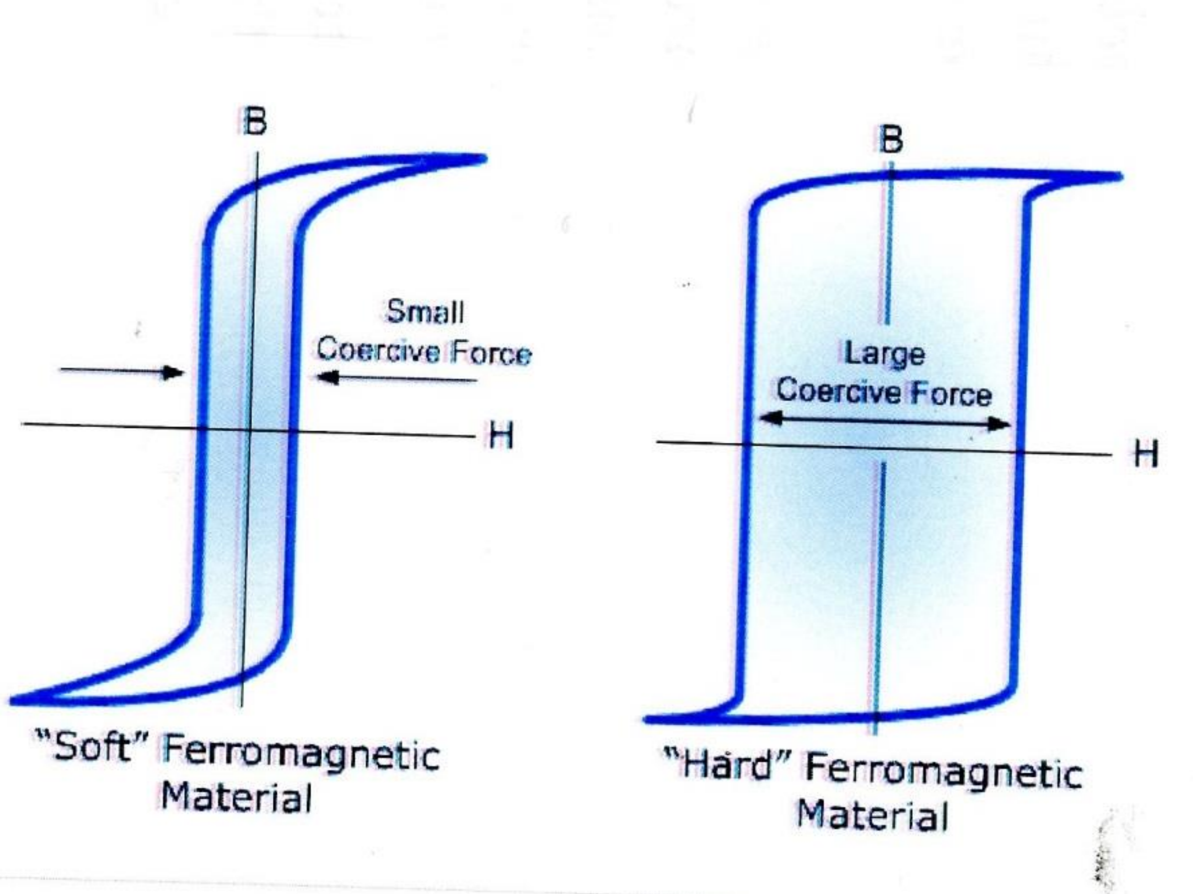
(Received 10 March 1971 by T. Muto)

Solid State Communications

vol. 9, p. 705-707, 1971.



Researchers were *mystified* by *hysteresis loops* which pass through the *origin* !



Magnetic Hysteresis

The lag or delay of a magnetic material known commonly as *Magnetic Hysteresis*, relates to the magnetization properties of a material by which it firstly becomes magnetized and then demagnetized.

Then Research was Frozen for the next 30 years !

Less than 10 papers on ***Solid state non-volatile memory devices***

were published between

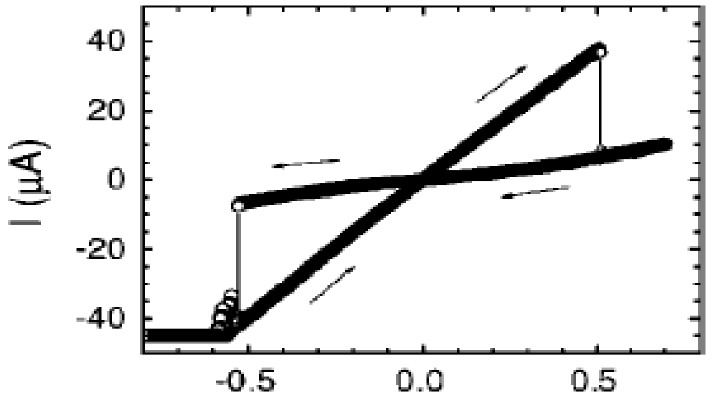
1970 and ***2000*** **!**

55 Years of ***confused and misunderstood*** non-volatile memory Devices

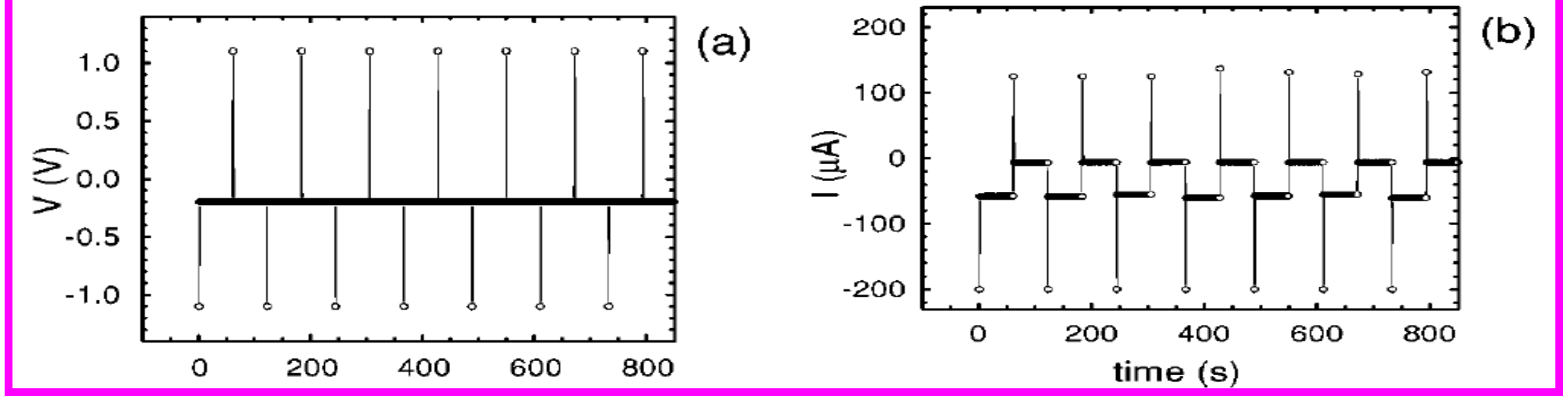
APPLIED PHYSICS LETTERS VOLUME 77, NUMBER 1 3 JULY 2000

Reproducible switching effect in thin oxide films for memory applications

A. Beck, J. G. Bednorz, Ch. Gerber, C. Rossel,[a)] and D. Widmer
IBM Research, Zurich Research Laboratory, CH—8803 Rüschlikon, Switzerland



Pinched Hysteresis loop of Cr-doped $SrZrO_3$ memristor



Switching performance of a capacitor-like structure based on Cr-doped $SrZrO_3$. (a) Applied voltage vs time and (b) readout current vs time.

The switching operation of a Cr-doped $SrZrO_3$ device in the pulse mode is illustrated in Fig. 2. A negative voltage pulse of 2 ms switches the system into the low-impedance state. After applying a positive voltage pulse of 2 ms, the "information" written to the device is erased and the high-impedance state is recovered. Between each write and erase pulse the state is read every second for 1 min with 200 mV pulses of 2 ms duration. This switching behavior, which can be repeated reproducibly for longer periods, demonstrates the potential of such a simple capacitor-like structure to act as nonvolatile random access memory. In this example the write and erase voltages of ± 1.1 V are fairly small compared to those currently used in ferroelectric and FLASH memories and within the range of operation required in the future generations of microelectronic circuits. Faster switching speeds, i.e., shorter write and erase pulses as used here, are also possible but require higher voltage amplitudes. So far the fastest reproducible switching could be achieved with 100 ns write/erase pulses at an amplitude of ± 5 V. In our experi-

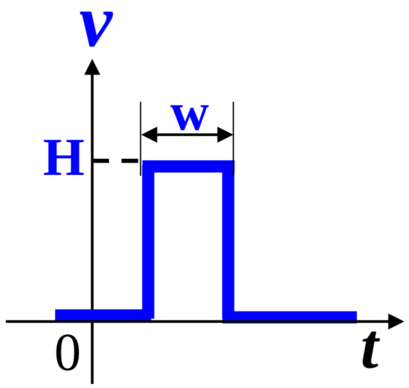
A negative voltage pulse of 2 ms switches the system into the low-impedance state. After applying a positive voltage pulse of 2 ms, the "information" written to the device is erased and the high impedance state is recovered.

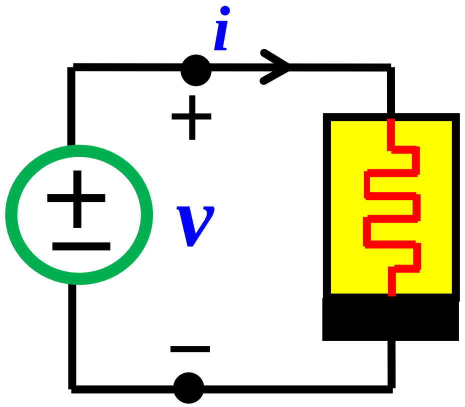
Faster switching speeds, i.e., shorter write and erase pulses as used here, are also possible but require higher voltage amplitudes.

1987
Nobel
Prize
in
Physics

Discovers
high-
temperature
Super-
conductors

Johannes Georg Bednorz

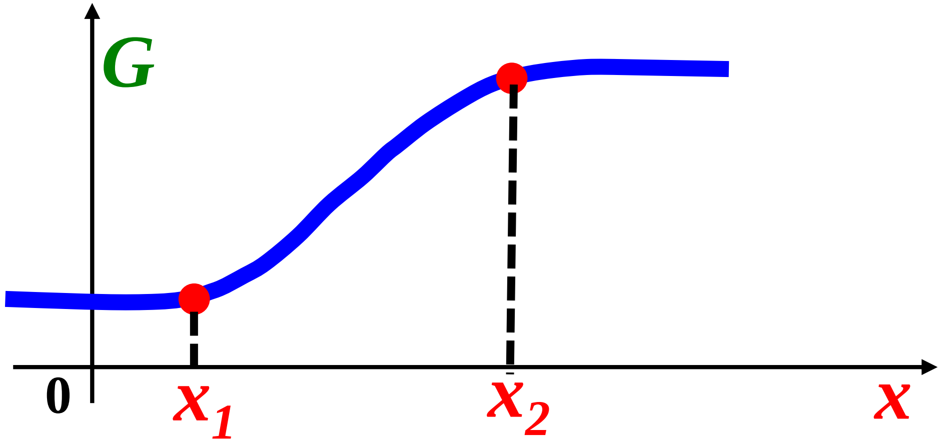




$$i = G(x)v$$

$$\dot{x} = f(x, v)$$

$$f(x, 0) = 0$$

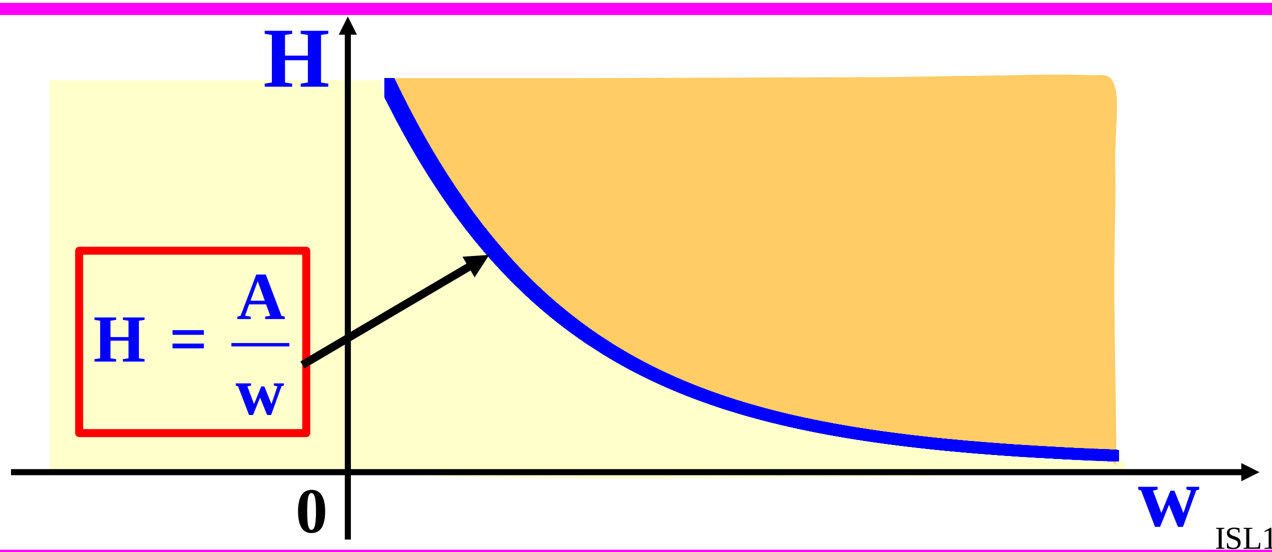


Minimum Pulse-Area Switching Theorem

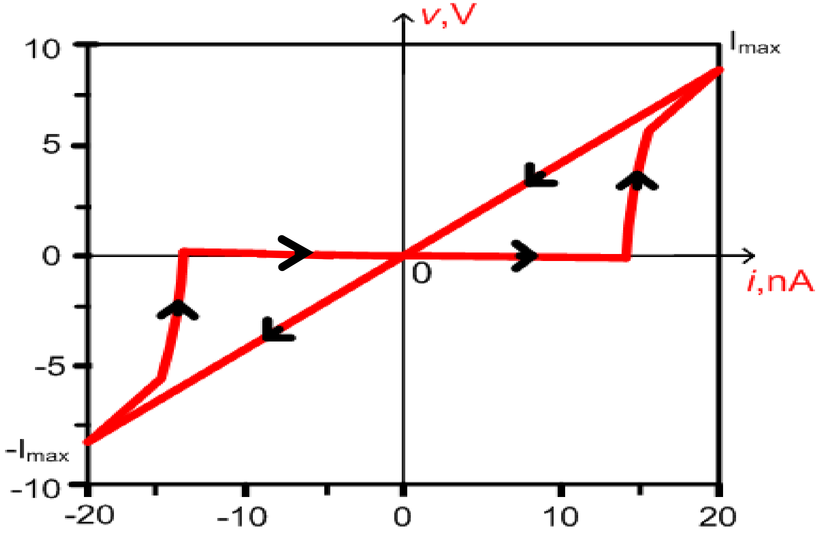
There is a *minimum pulse area* $A(x_1, x_2)$

$$w \times H = A(x_1, x_2)$$

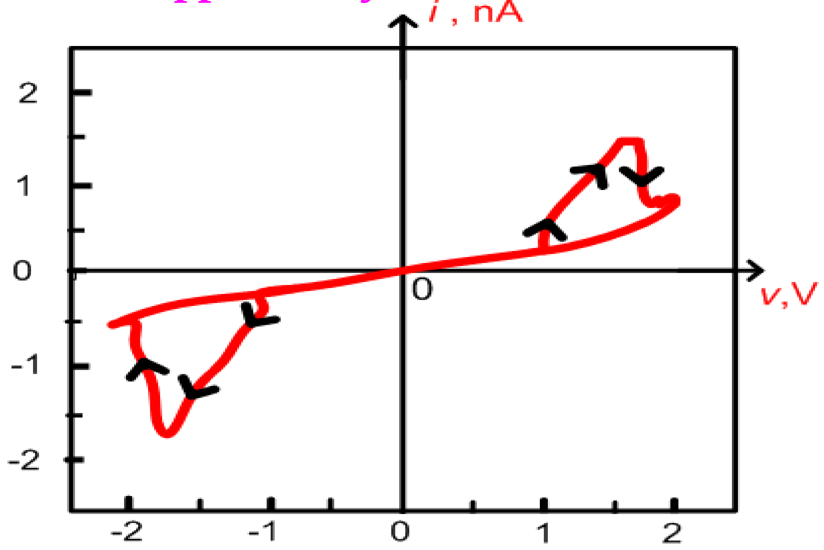
to switch between any *2* states x_1 and x_2 of any *non-volatile memristor*.



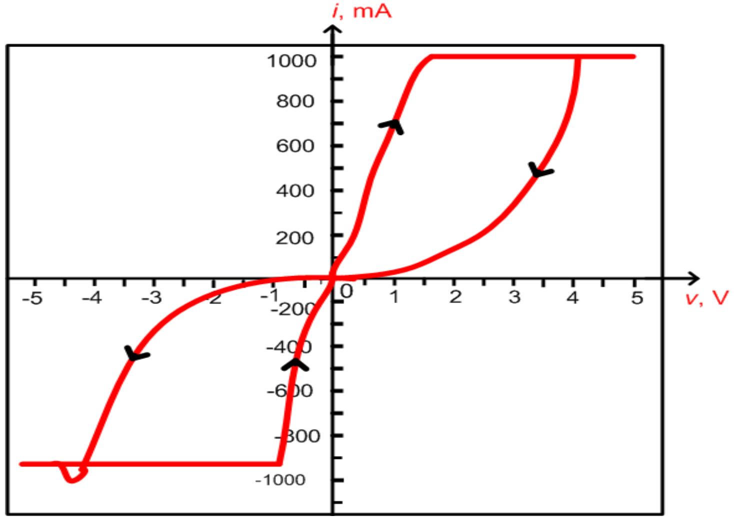
(a) *2009, Kim, et al., Nano Letters*



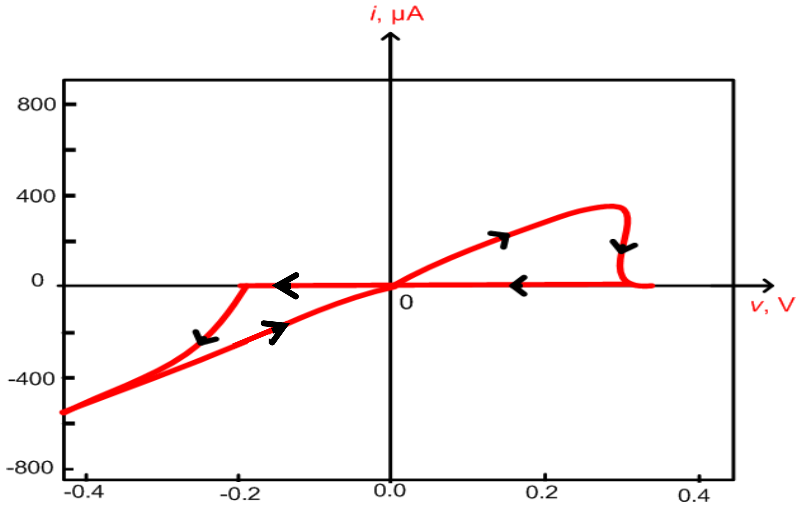
(b) *2012, Yang, et al., Applied Physics Letters*



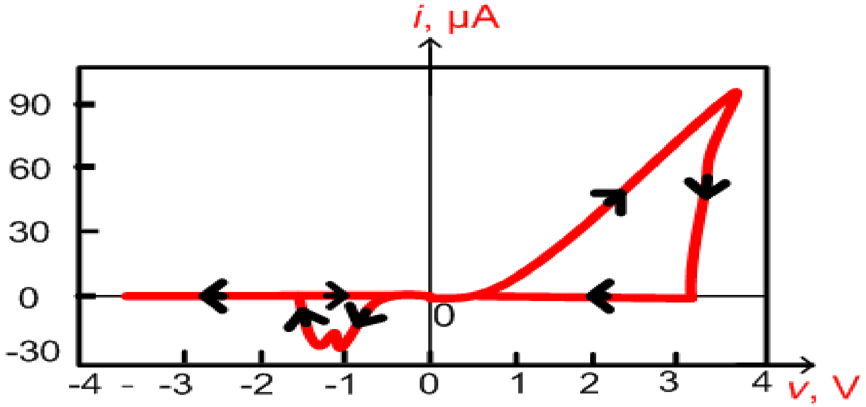
(c) *2011, Szot, et al., Nanotechnology*



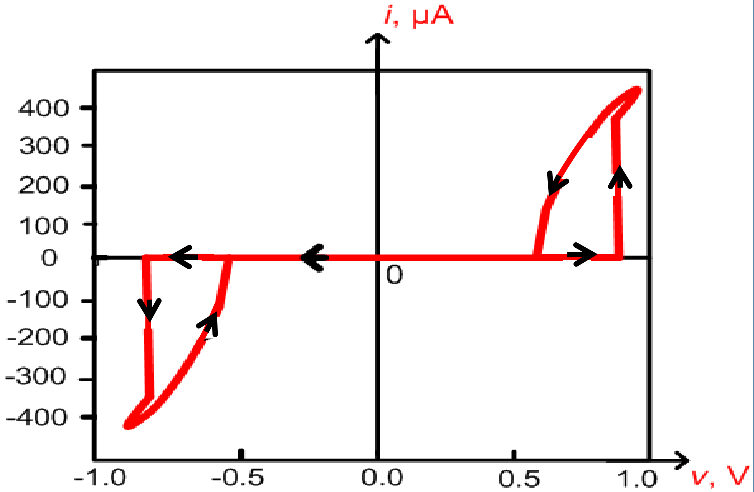
(d) *2011, Hino, et al., Sci. Technol. Adv. Mater.*

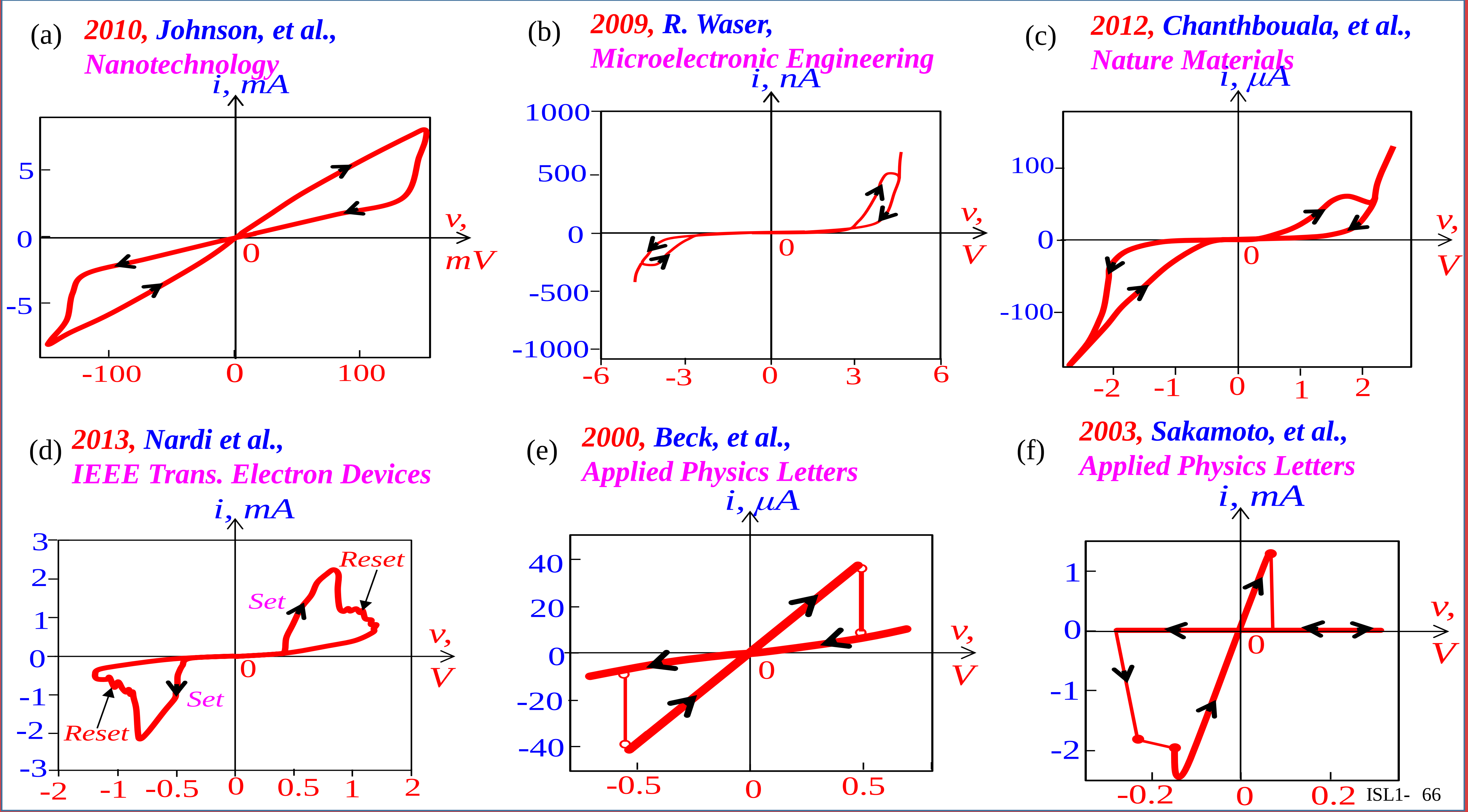


(e) *2009, Jo, et al., Nano Letters*



(f) *2012, Pickett , et al., Nature Materials*




(a) 2010, Johnson, et al., Nanotechnology
i, mA
5
0
-5
-100
0
100
v, mV
(b) 2009, R. Waser, Microelectronic Engineering
i, nA
1000
500
0
-500
-1000
-6
-3
0
3
6
v, V
(c) 2012, Chanthbouala, et al., Nature Materials
i, μA
100
0
-100
-2
-1
0
1
2
v, V
(d) 2013, Nardi et al., IEEE Trans. Electron Devices
i, mA
3
2
1
0
-1
-2
-3
-2
-1
-0.5
0
0.5
1
2
Reset
Set
Set
Reset
v, V
(e) 2000, Beck, et al., Applied Physics Letters
i, μA
40
20
0
-20
-40
-0.5
0
0.5
v, V
(f) 2003, Sakamoto, et al., Applied Physics Letters
i, mA
1
0
-1
-2
-0.2
0
0.2
v, V

Not all

memristors are

Non-Volatile !

Non-Volatile Memristor

Theorem* : *Two implies Infinite !

If a *passive memristor* exhibits **2** *stable memory states*, then it has a *continuum* of *stable memory* states.

Fundamental memristor memory Theorem

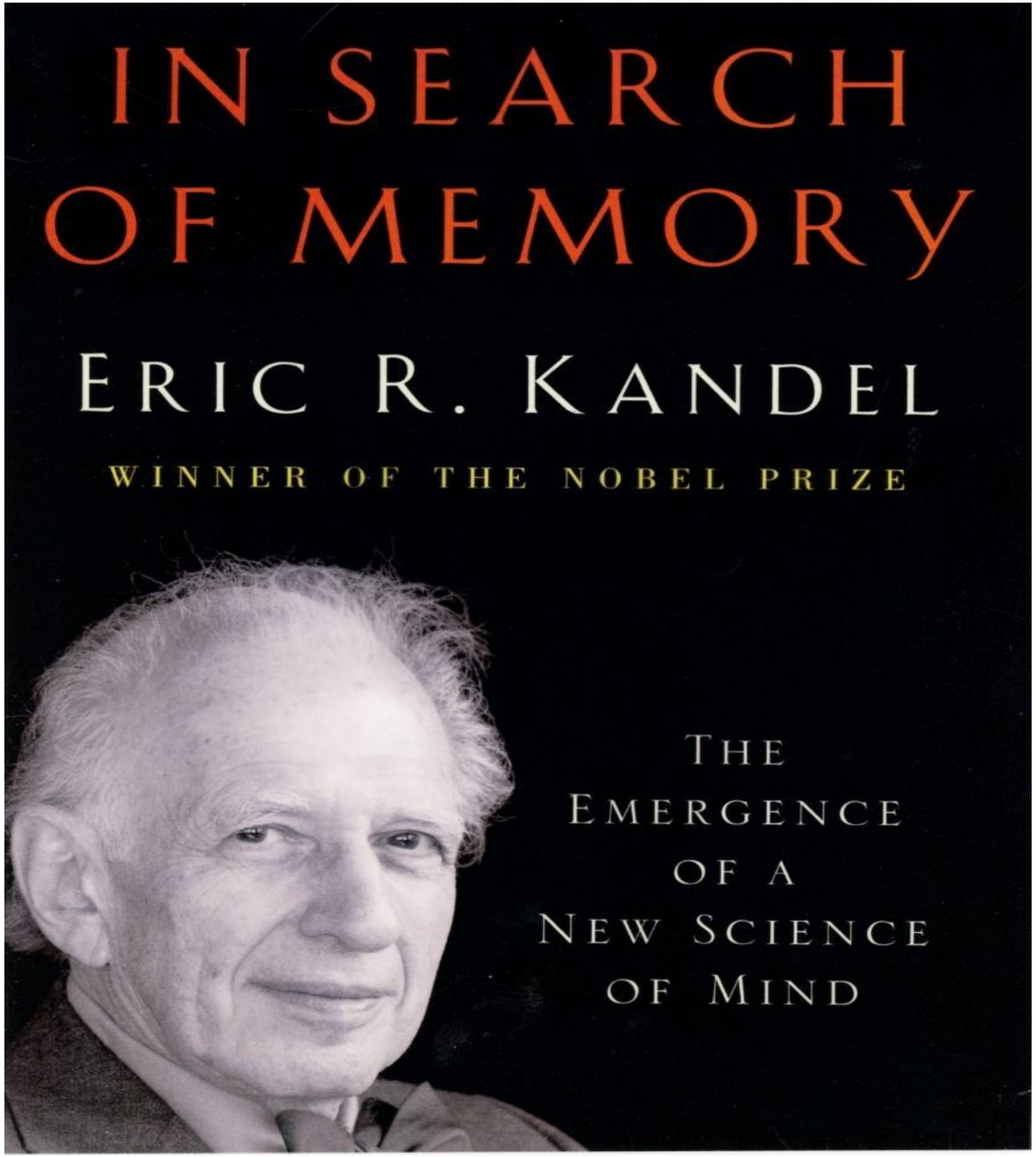
All passive non-volatile memristor memories

are

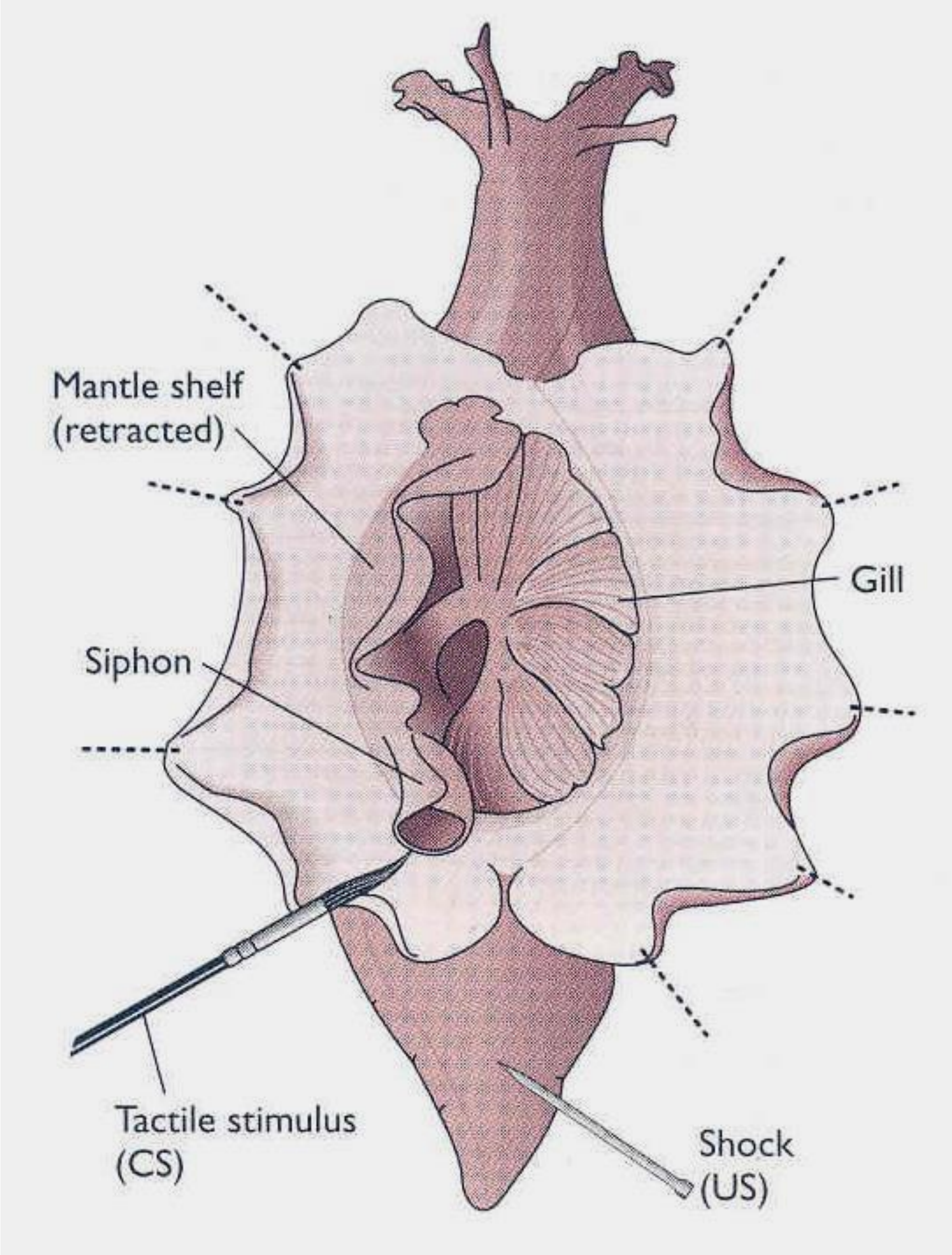
continuum (analog) memories.

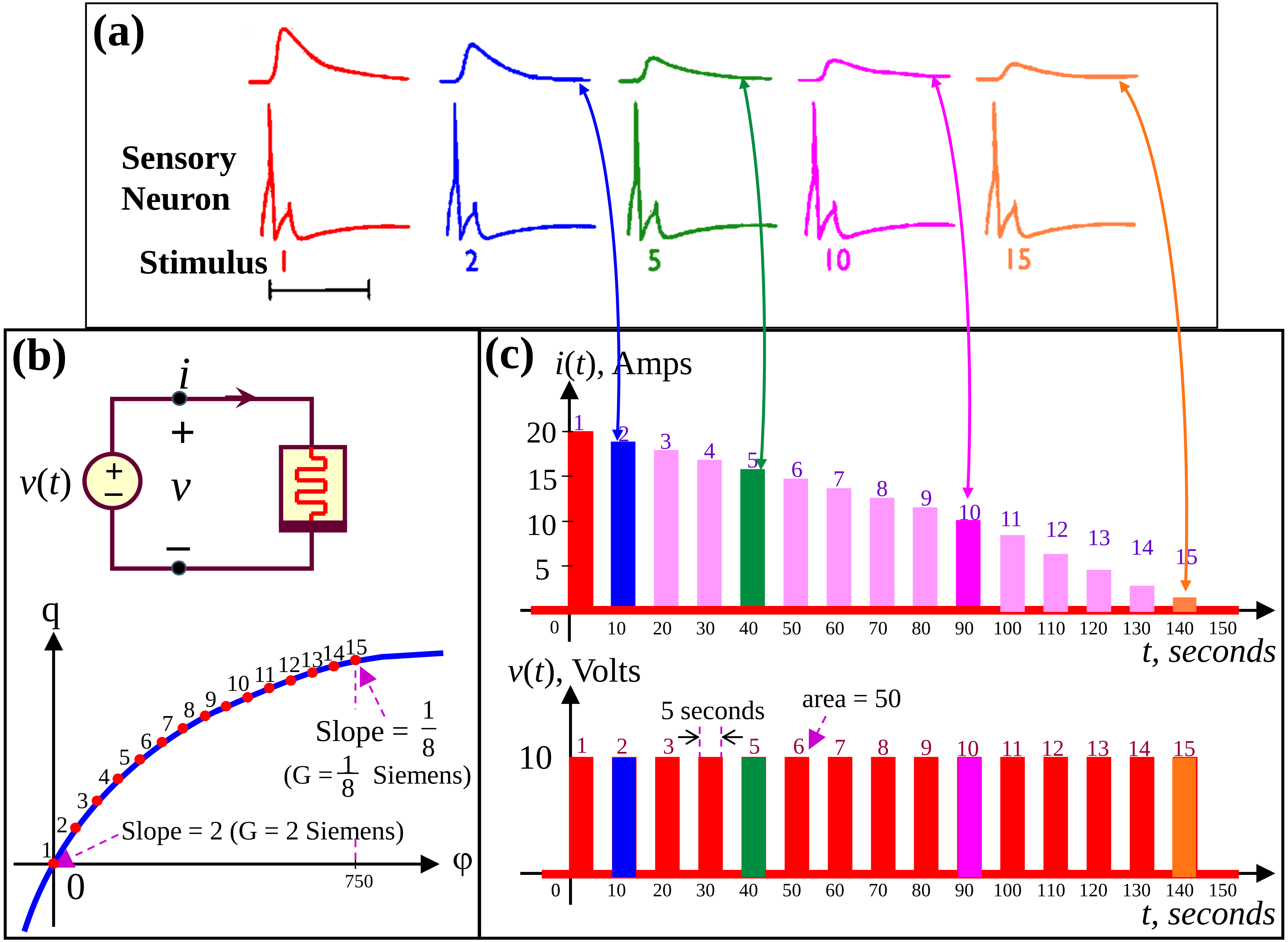
Aplysia* with a *Nobel Prize Medal

2000 Nobel Prize in Physiology

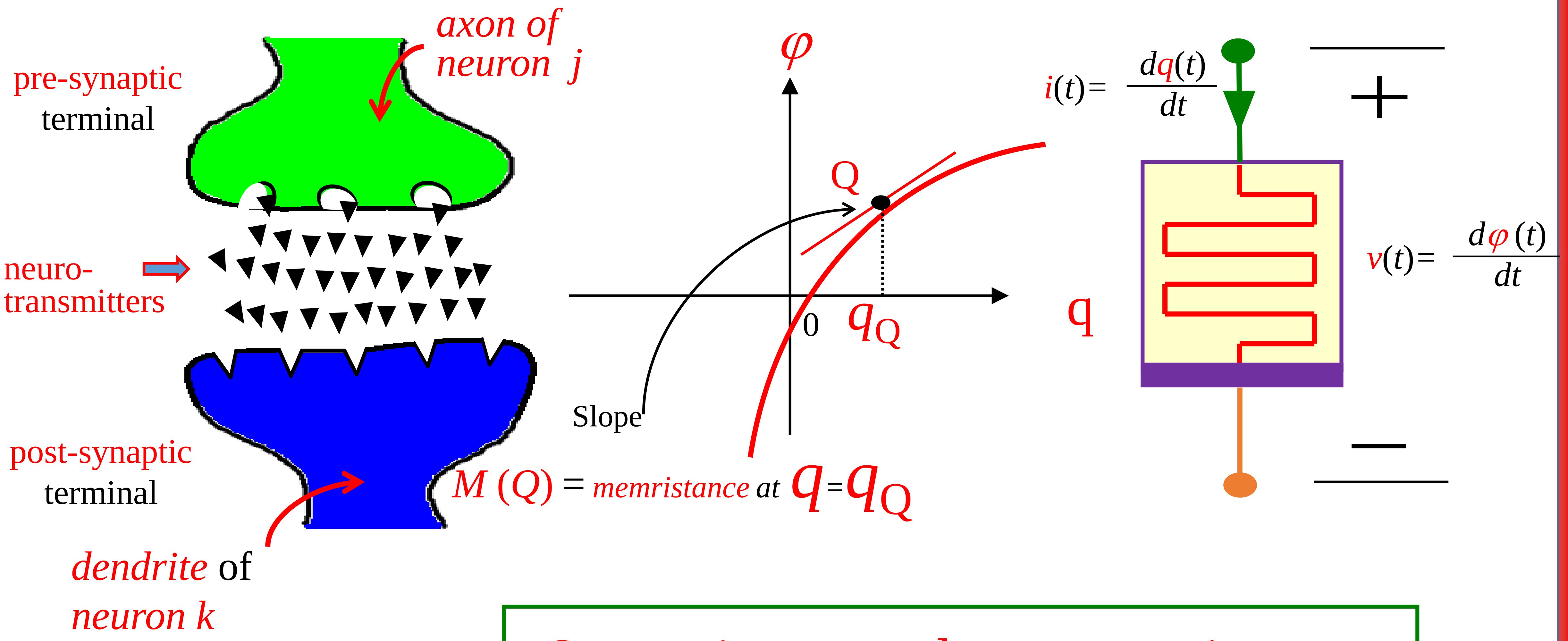


Discovers the molecular basis of memory in Aplysia


Mantle shelf
(retracted)
Gill
Siphon
Tactile stimulus
(CS)
Shock
(US)


(a)
Sensory
Neuron
Stimulus
1
2
5
10
15
(b)
i
+
v(t)
v
−
q
φ
0
750
Slope = 1/8
(G = 1/8 Siemens)
Slope = 2 (G = 2 Siemens)
(c)
i(t), Amps
t, seconds
v(t), Volts
5 seconds
area = 50
t, seconds

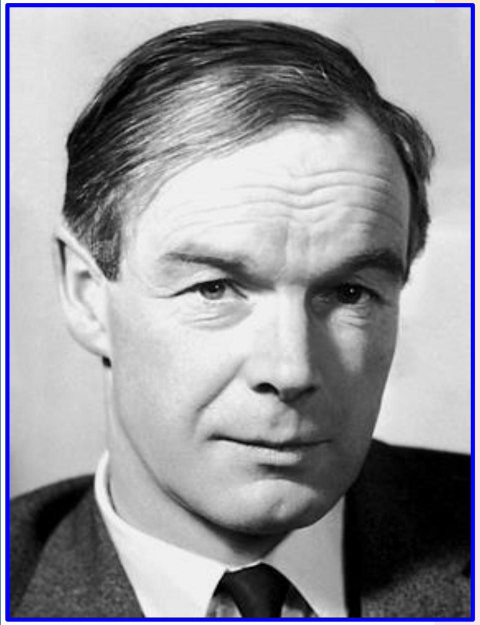
Synapses are Memristors



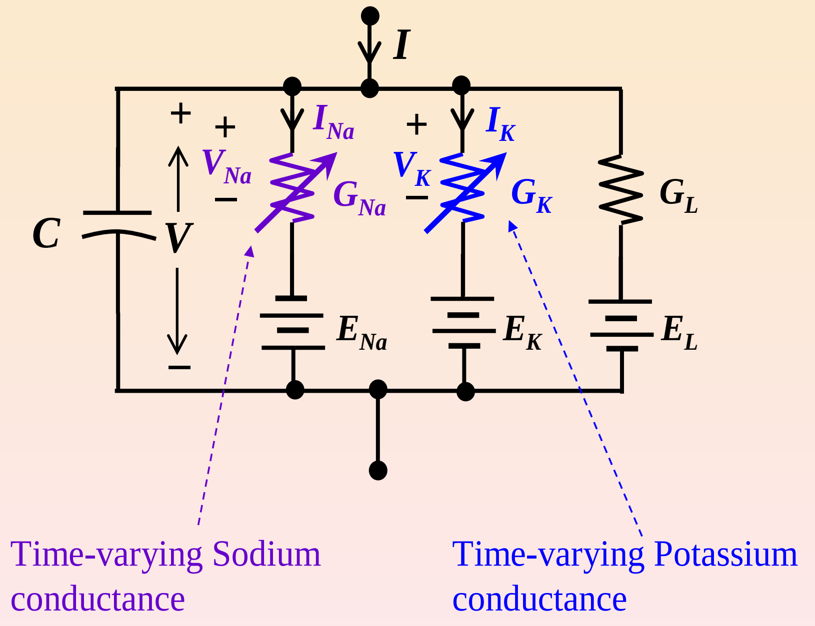
Synaptic strength ≡ memristance

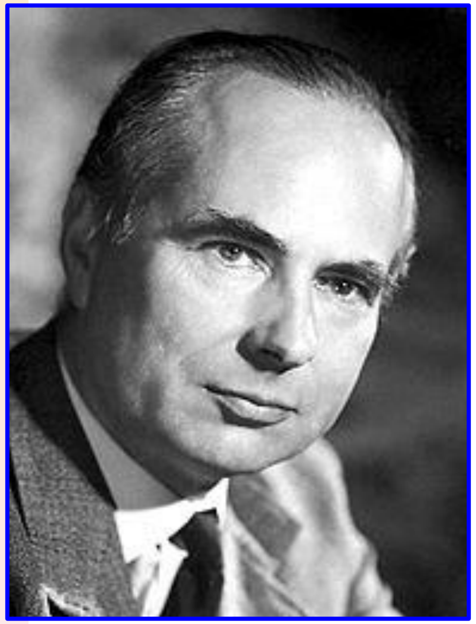
1961 Nobel Prize in Physiology

Hodgkin- Huxley Nerve Membrane Model



Sir A. L. Hodgkin





Sir A. F. Huxley

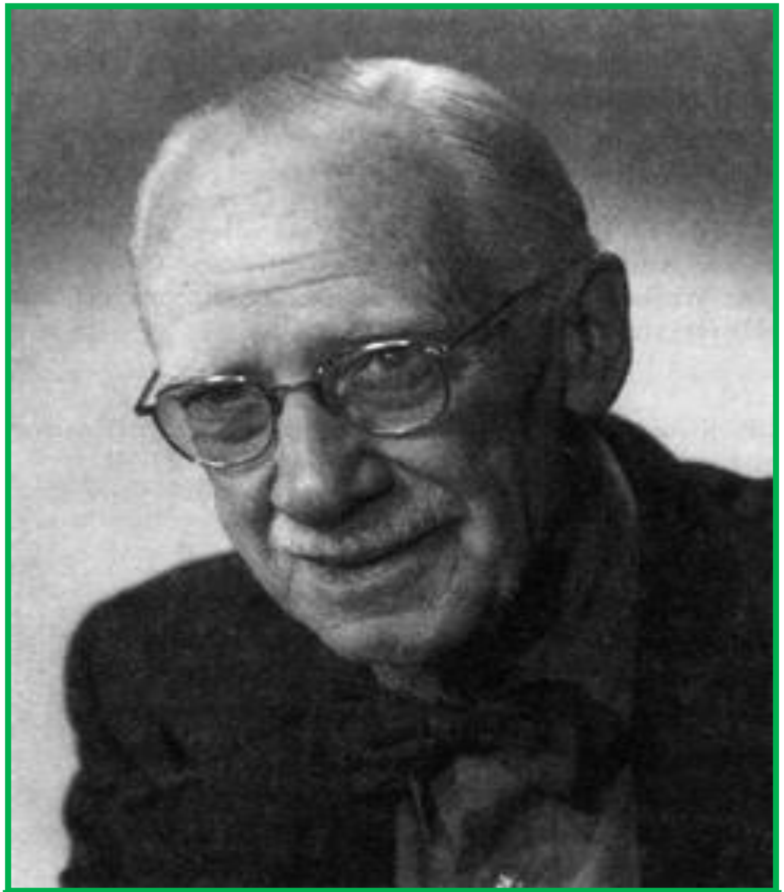
From

A.L. Hodgkin and A. F. Huxley

A Quantitative Description of Membrane Current and its Application to Conduction and Excitation in Nerve.

Journal of Physiology, Vol. 117, pp.500-544, 1952

The suggestion of an inductive reactance anywhere in the system was shocking to the point of being unbelievable.



Kenneth Cole

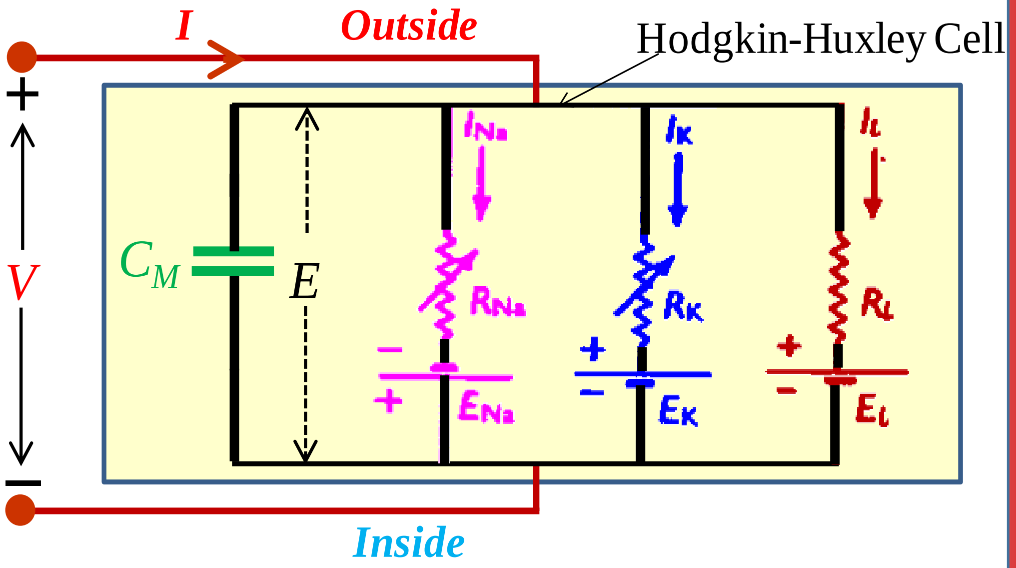
Hodgkin's Blunder

Hodgkin had struggled in vain searching for a *physical* interpretation of the *squid axon inductance*. He failed because he had mistaken the *axon* for a *time-varying conductance*, when in fact it has a simple explanation if the *Potassium* and *Sodium ion channels* are identified as *memristors*.

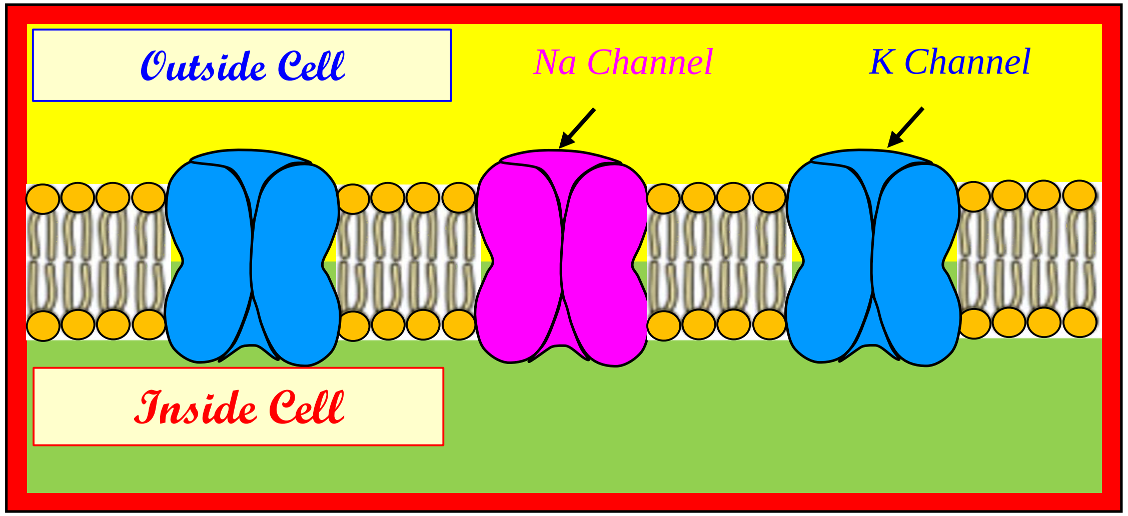
A. L. Hodgkin,
"The ionic basis of electrical activity in nerve and muscle,"
Biological Review, Vol. 26, pp. 339-409, 1951

(a)

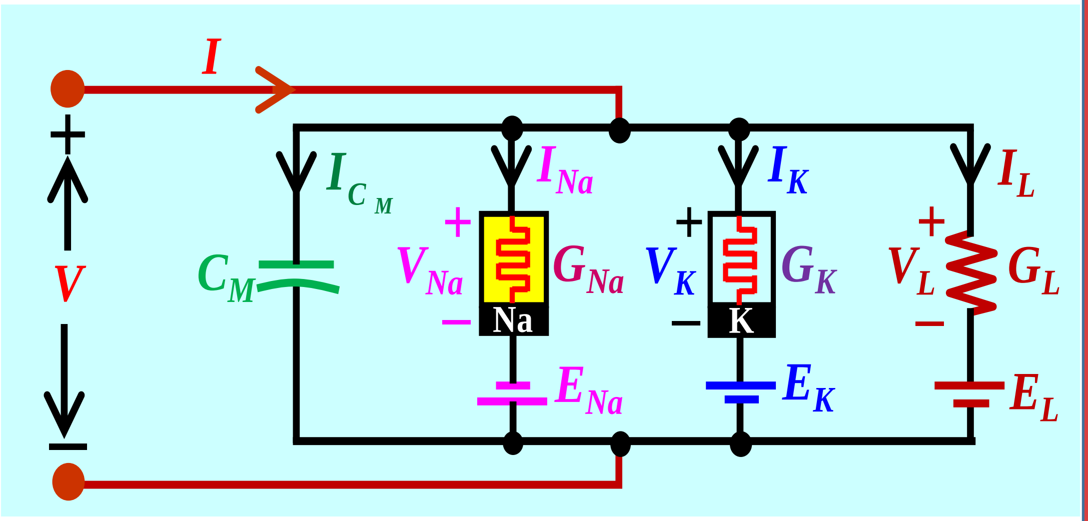
(c)


I
Outside
Hodgkin-Huxley Cell
+
V
−
C_M
E
I_{Na}
R_{Na}
E_{Na}
I_K
R_K
E_K
I_L
R_L
E_L
Inside

(b)


Outside Cell
Na Channel
K Channel
Inside Cell

(d)

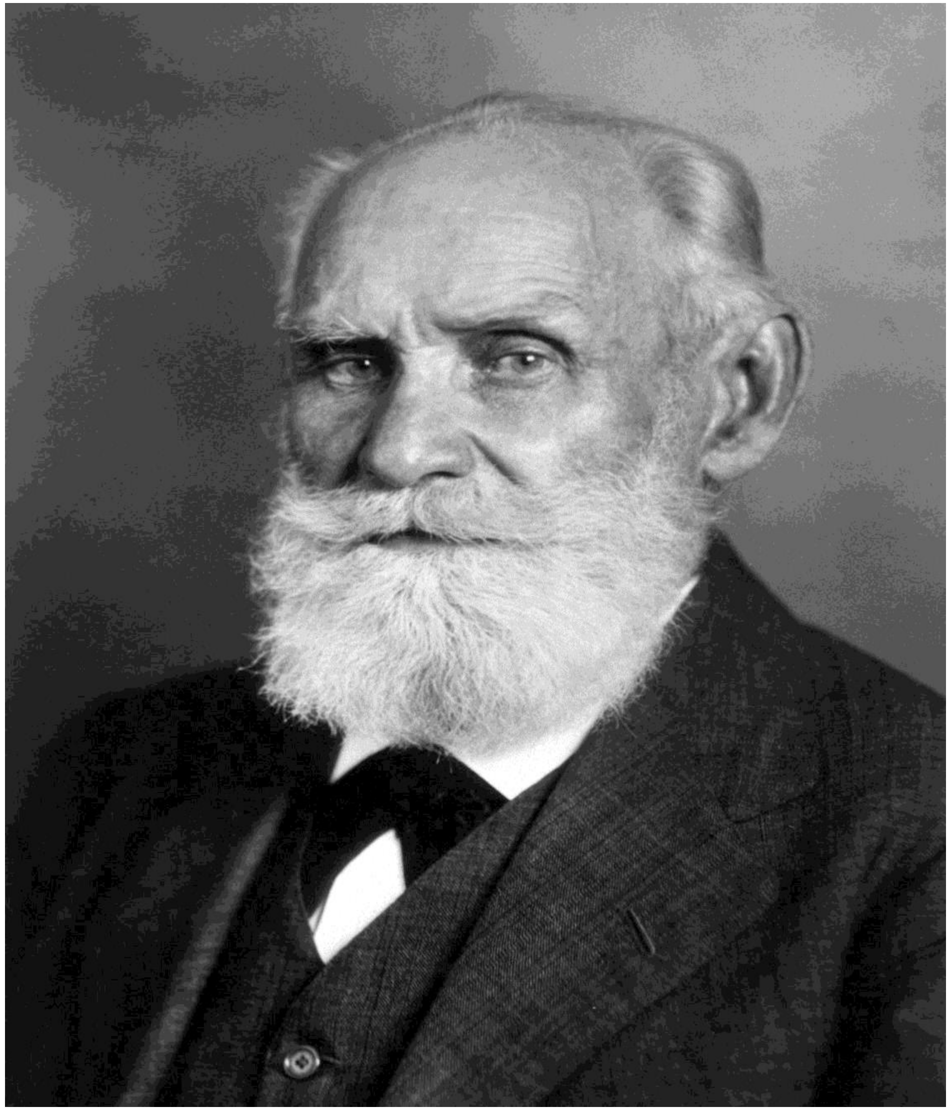

I
+
V
−
I_{C_M}
C_M
I_{Na}
V_{Na}
G_{Na}
Na
E_{Na}
I_K
V_K
G_K
K
E_K
I_L
V_L
G_L
E_L

Axons

are made of

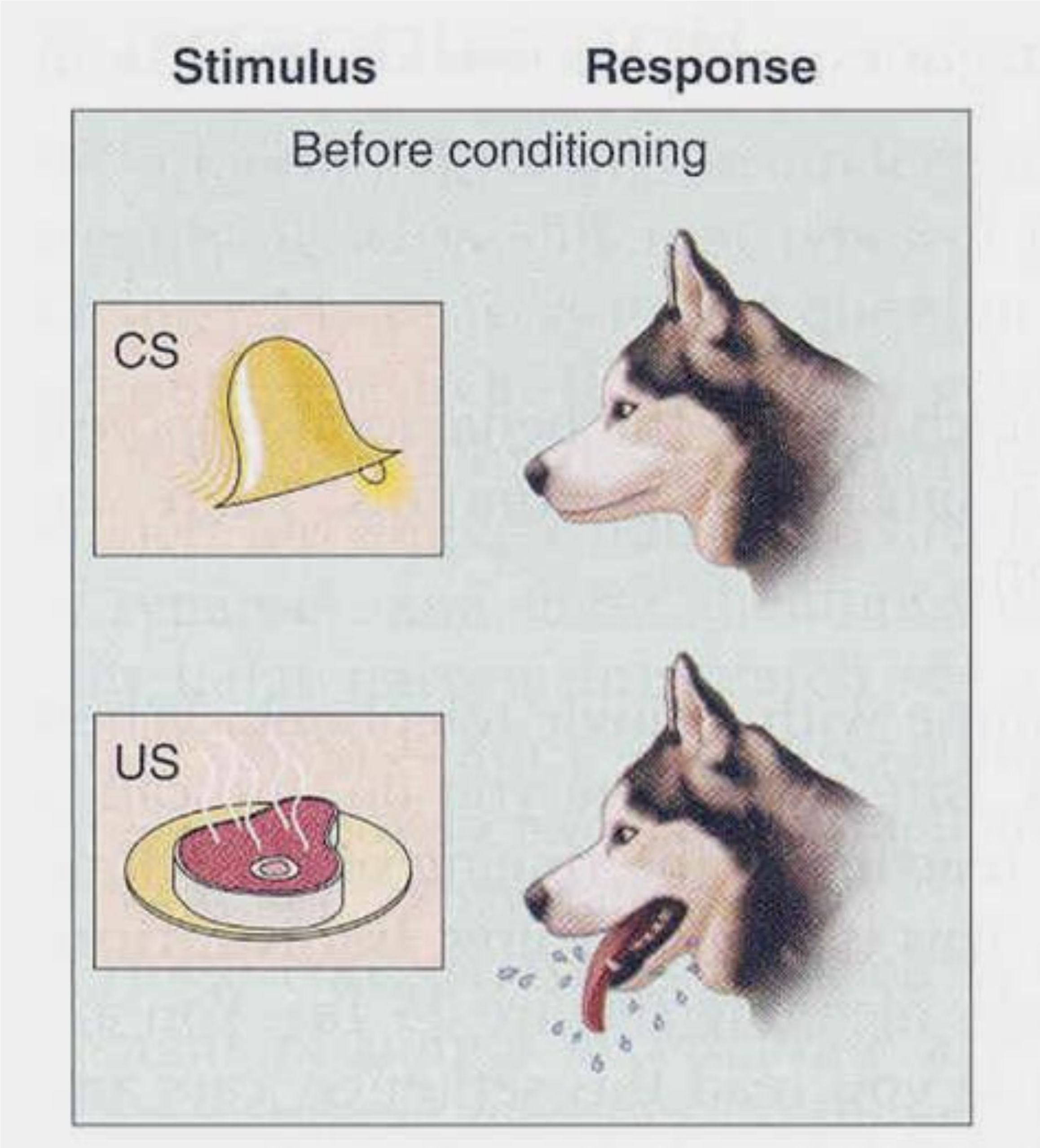
Memristors !

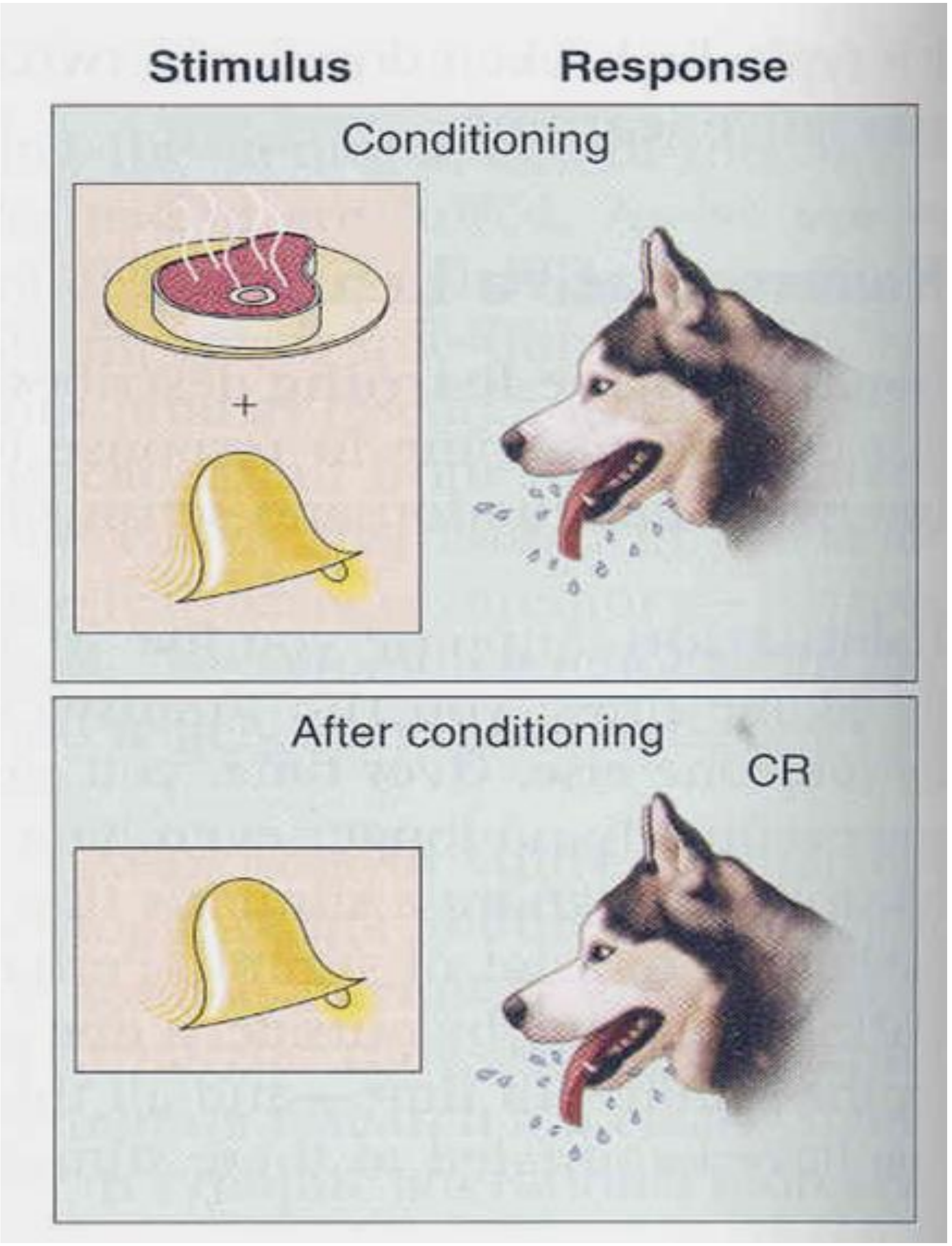
1904
Nobel
Prize
in
Physiology



Ivan P. Pavlov

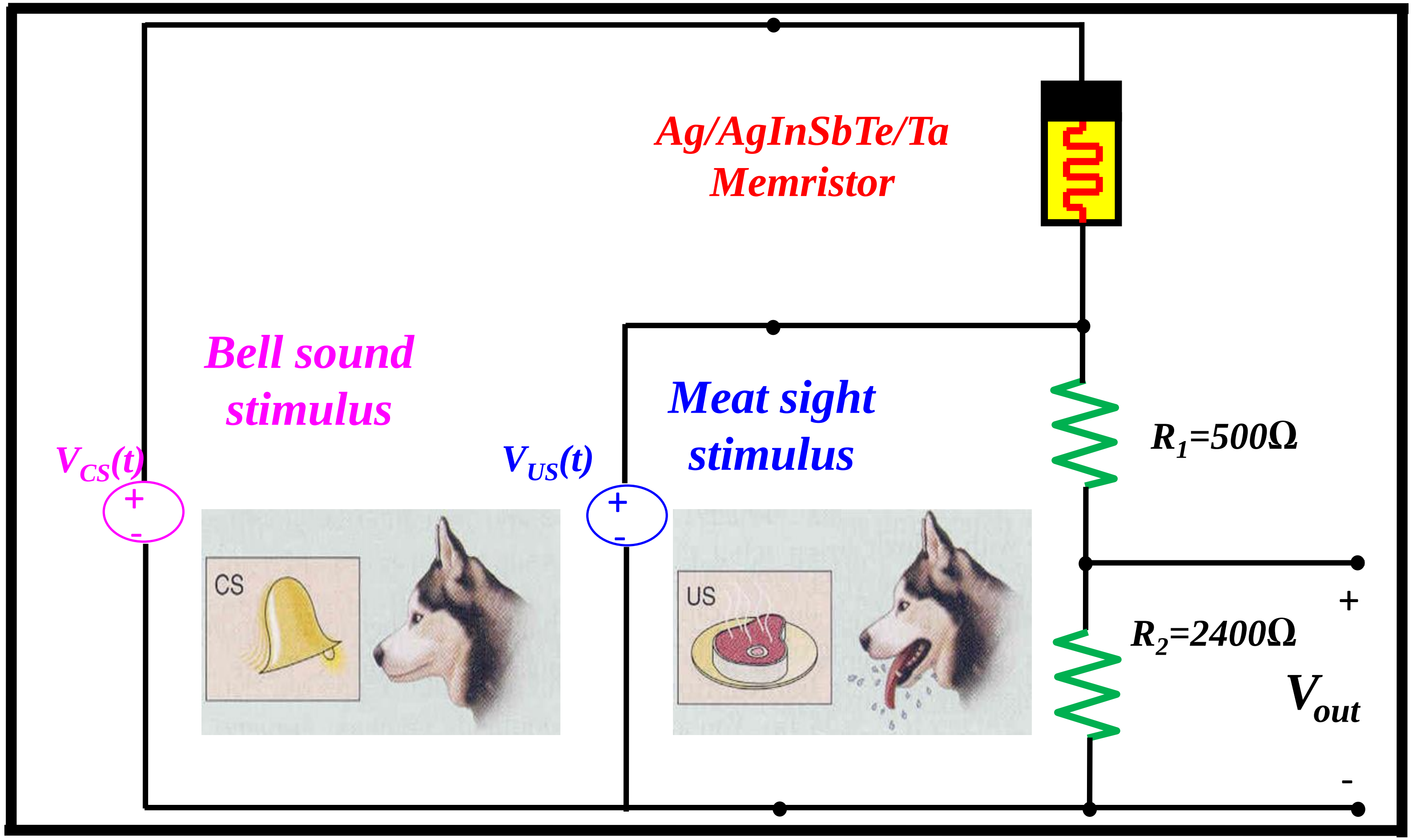
Discovers
Associative
memory
and
learning
phenomenon


Stimulus
Response
Before conditioning
CS
US


Stimulus
Response
Conditioning
+
After conditioning
CR

Emulating Pavlov's Dog Associative Learning Phenomenon

Memristor Circuit for Emulating Pavlov's Dog



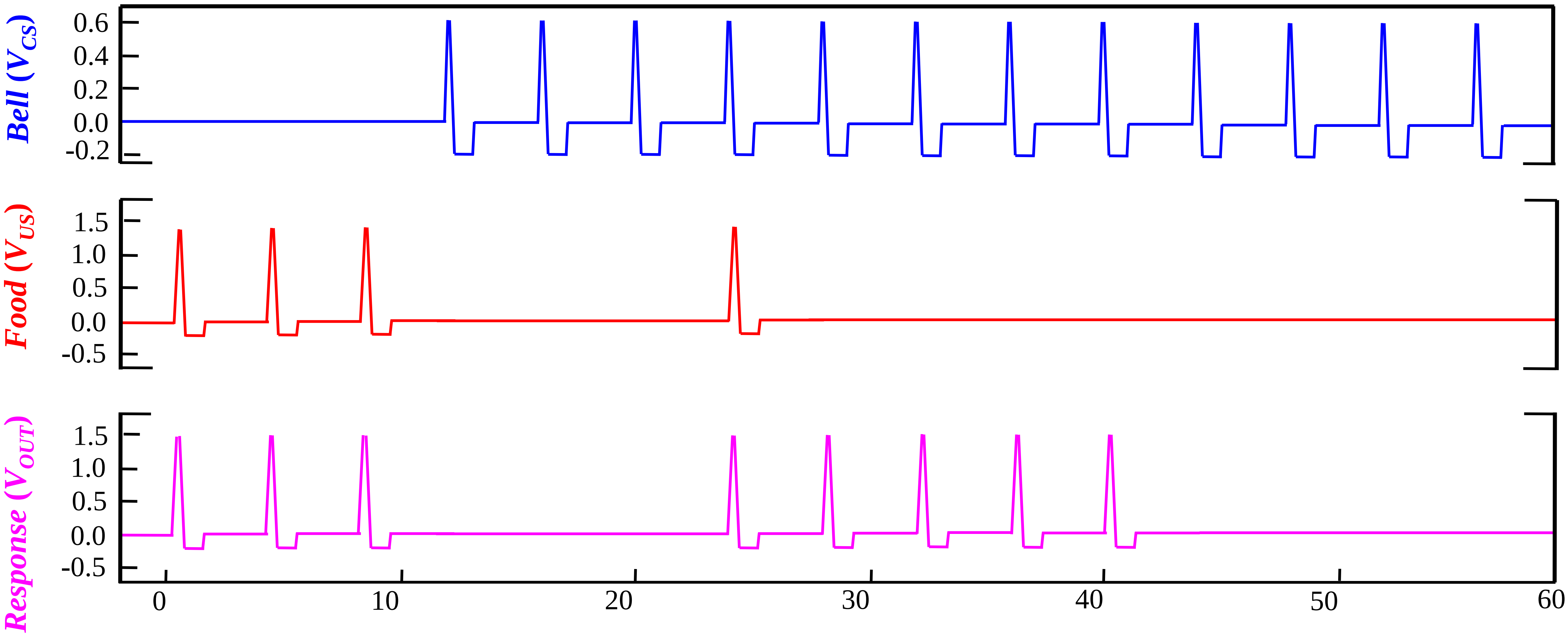
From:

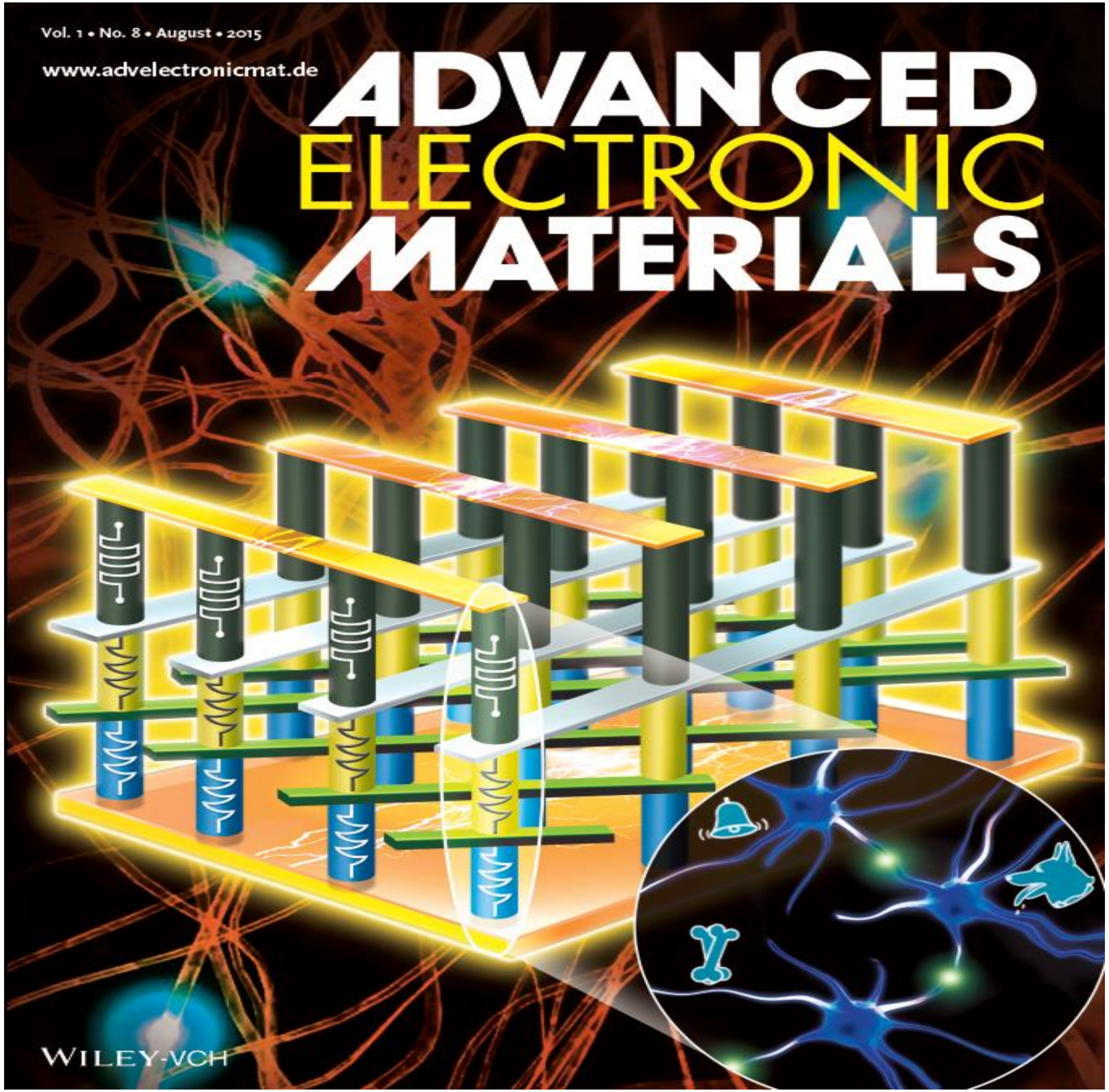
Y. Li, L. Xu, Y. P. Zhong, Y. X. Zhou, S. J. Zhong, Y. Z. Hu, L. O. Chua, X. S. Miao

Associative Learning with Temporal Contiguity in a Memristive Circuit for Large-Scale Neuromorphic Networks

Adv. Electronic Materials, 1500125, p.1-8,2015.

Waveforms Measured from Memristor Circuit Emulating Pavlov's Experiment




Vol. 1 • No. 8 • August • 2015
www.advelectronicmat.de
ADVANCED
ELECTRONIC
MATERIALS
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A Medieval Catapult

K. Liu, C. C. Cheng, J. Suh,
R. T.Kong. D. Fu, S. Lee, J. Zhou,
L. O. Chua and J. Wu

Advanced Materials

Vol.26,no.11,pp.1746-1750

March 2014

Dr. Junqiao Wu with President Obama

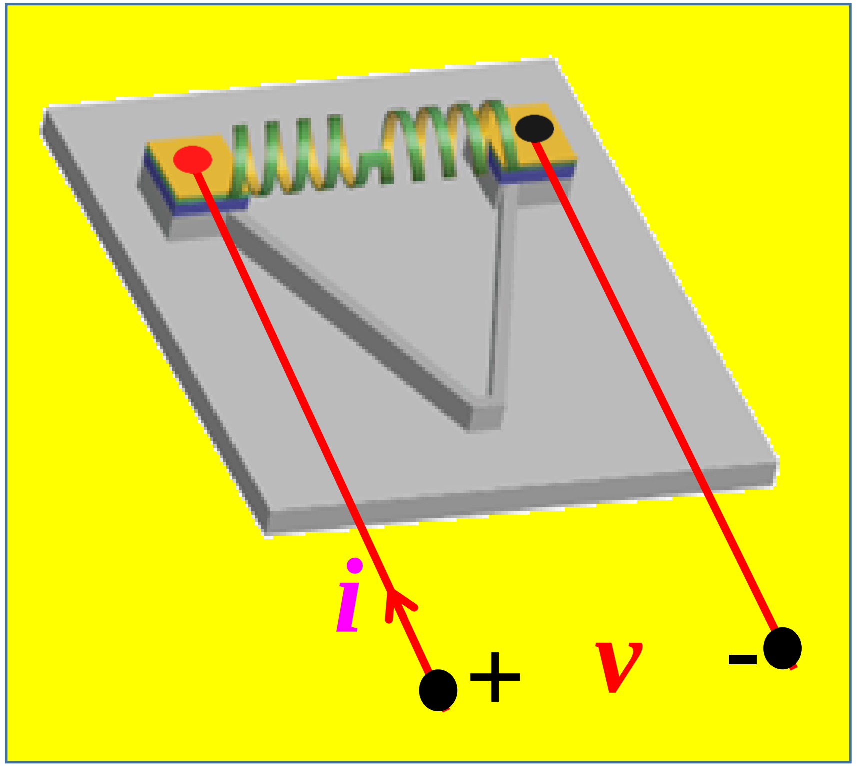
at the award ceremony for

Presidential Early Career Awards

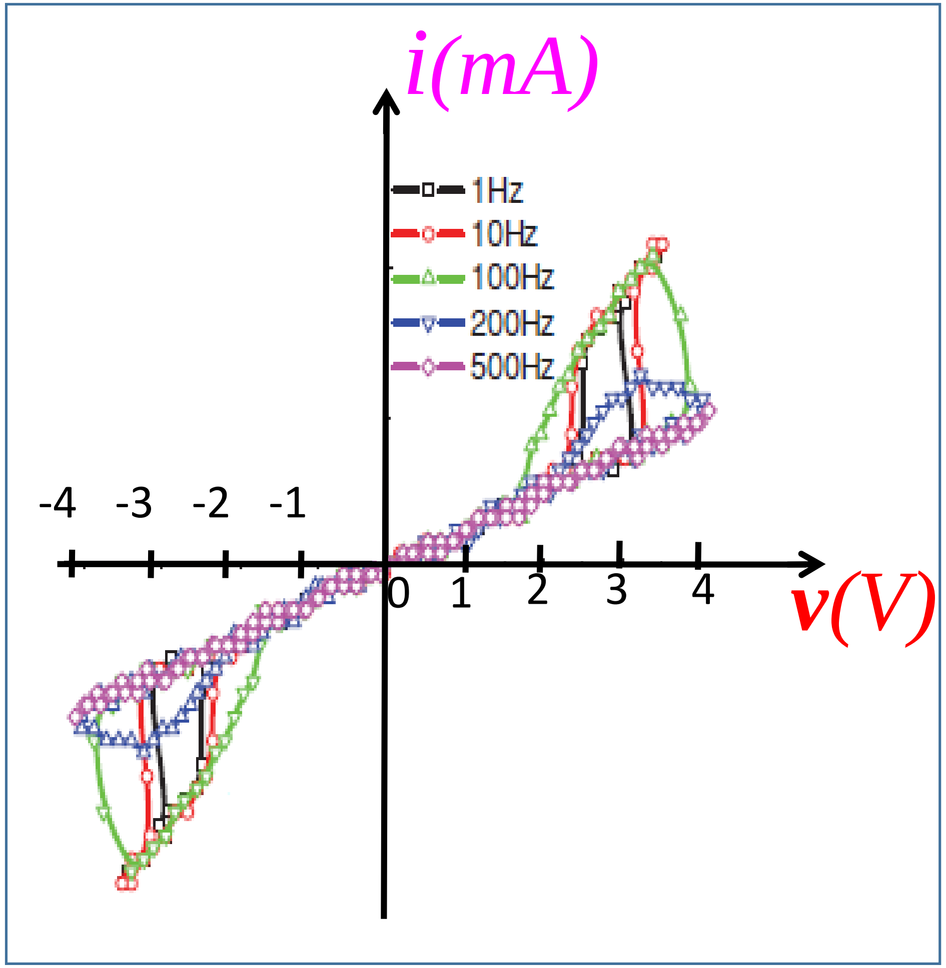
for Scientists and Engineers

Whitehouse, April 14, 2014.

A Micro-Catapult *Memristor*



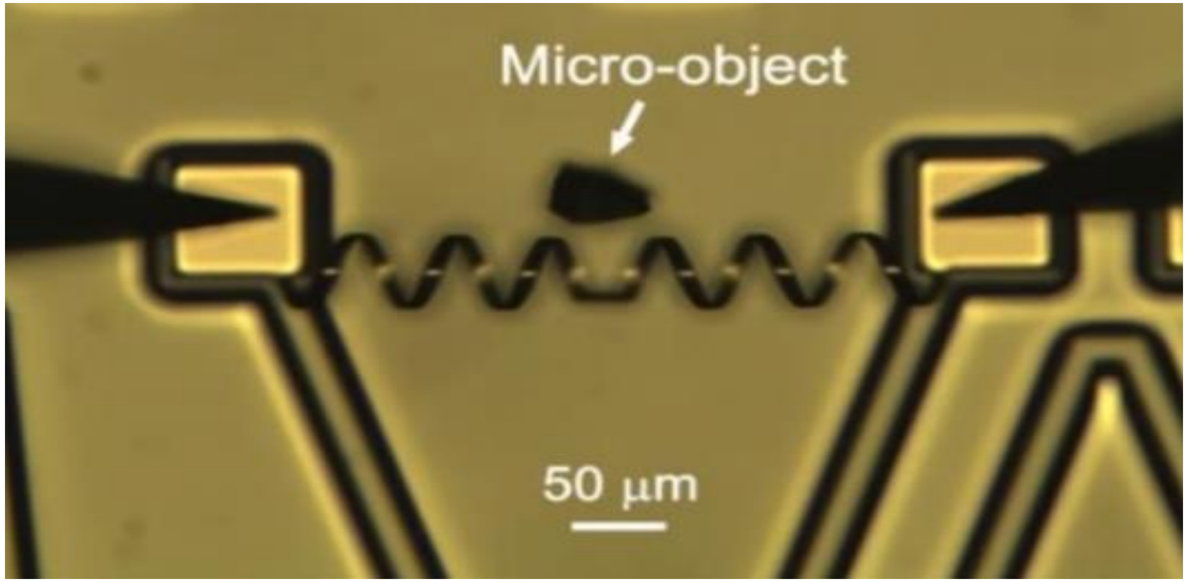
Micro-Catapult Memristor



Pinched Hysteresis Loop
Micro-Catapult Memristor

A Vanadium Dioxide Micro Catapult Memristor

From: Advanced Materials, 2013



50 μm Vanadium Dioxide *Memristor*

- 1000 times more powerful than a human muscle
- Can Catapult objects 50 times heavier than itself
- Can catapult objects over a distance 5 times its length
- Faster than the blink of an eye.

GE90 Turbofan for Boeing 777

Performance of Turbofan for Boeing 777:

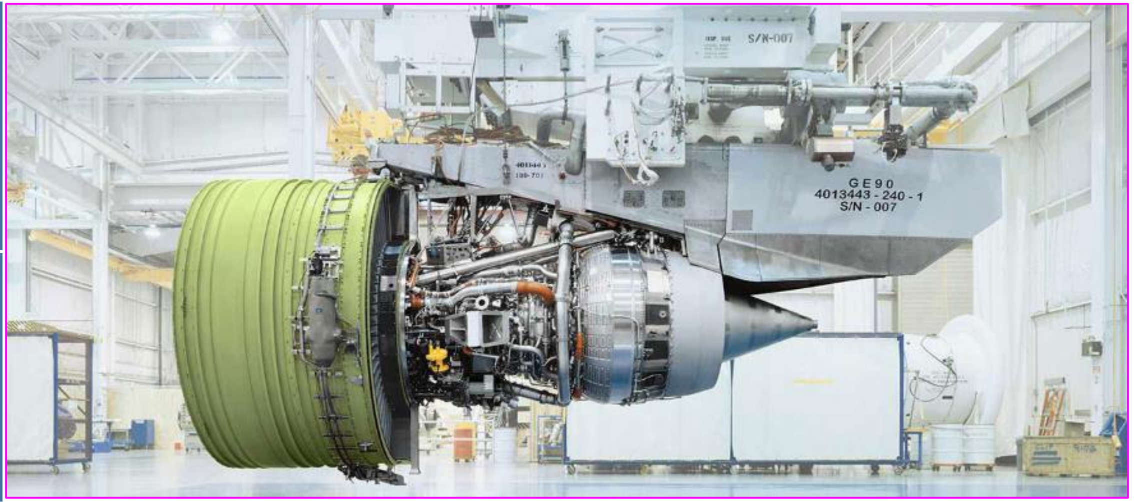
Power Density: 9 kW/kg

Rotation Speed: 10,000 rpm

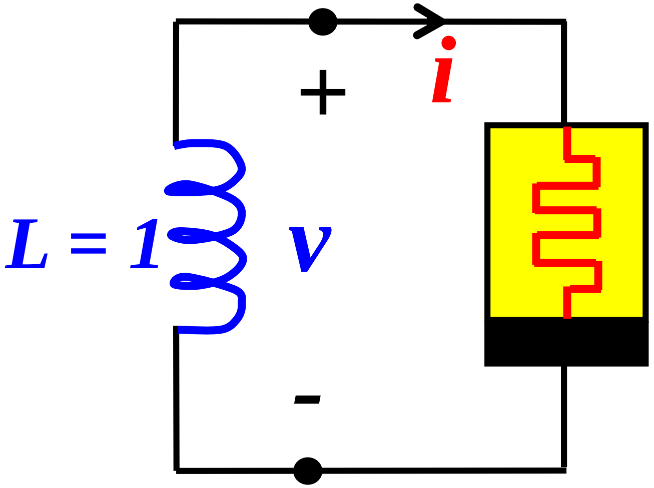
Performance of Micro Catapult Memristor :

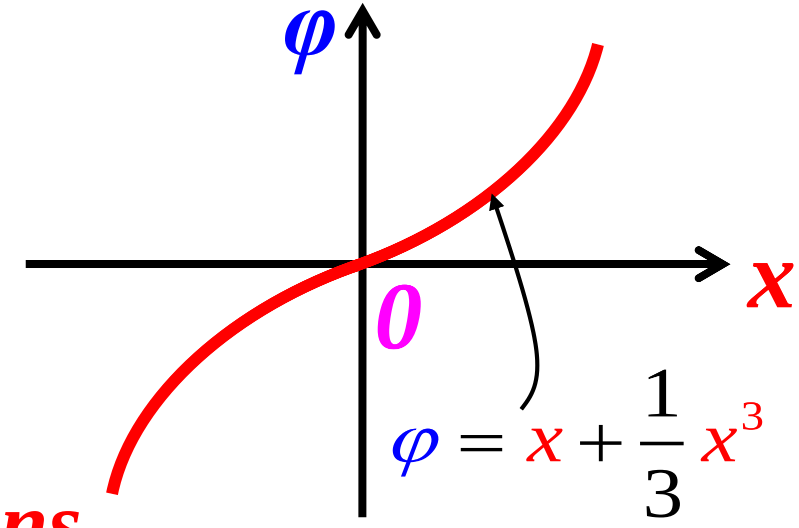
Power Density: 39 kW/kg

Rotation Speed: 200,000 rpm



Some Memristor Circuits have Hamiltonians !





State Equations

$$\frac{dx}{dt} = i$$

$$\frac{di}{dt} = -(1+x^2)i$$

Hamiltonian Equations

$$\frac{dx}{d\tau} = \frac{\partial H}{\partial i}$$

$$\frac{di}{d\tau} = -\frac{\partial H}{\partial x}$$

$$\frac{dx}{d\tau} = 1$$

$$\frac{di}{d\tau} = -(1+x^2)$$

$$d\tau \triangleq i\,dt$$

$$\frac{dx}{i\,dt} = 1$$

$$\frac{di}{i\,dt} = -(1+x^2)$$

From :

M. Itoh and L. Chua

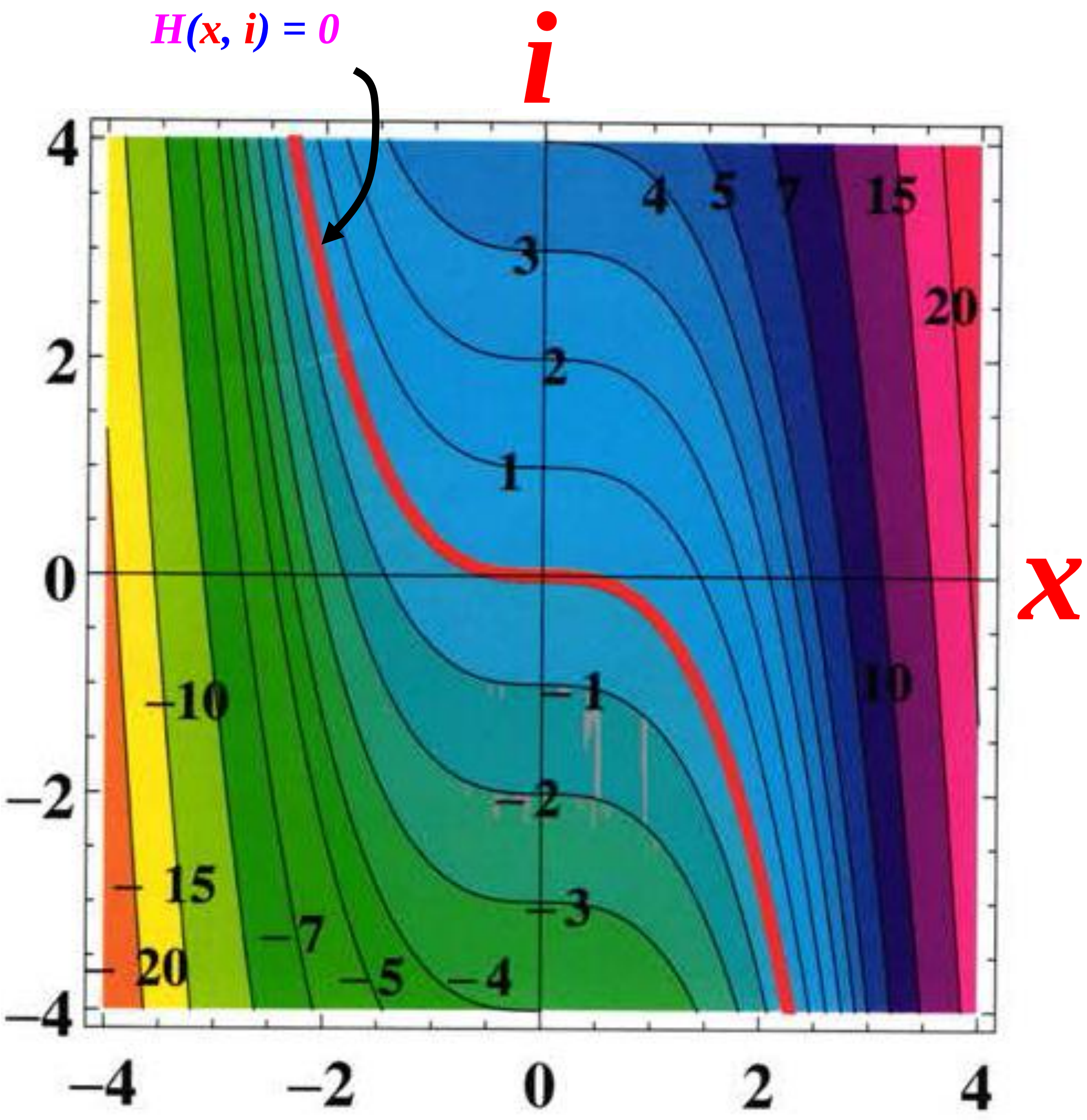
Memristor Hamiltonian Circuits

Int. J. of Bifurcation and Chaos

vol. 21, pp.2395-2425, 2011

Hamiltonian: $H(x,\, i) = \underbrace{(x + \frac{1}{3}x^3)}_{\text{Pseudo Potential Energy } H_x(x)} + \underbrace{i}_{\text{Pseudo Kinetic Energy } H_i(i)}$

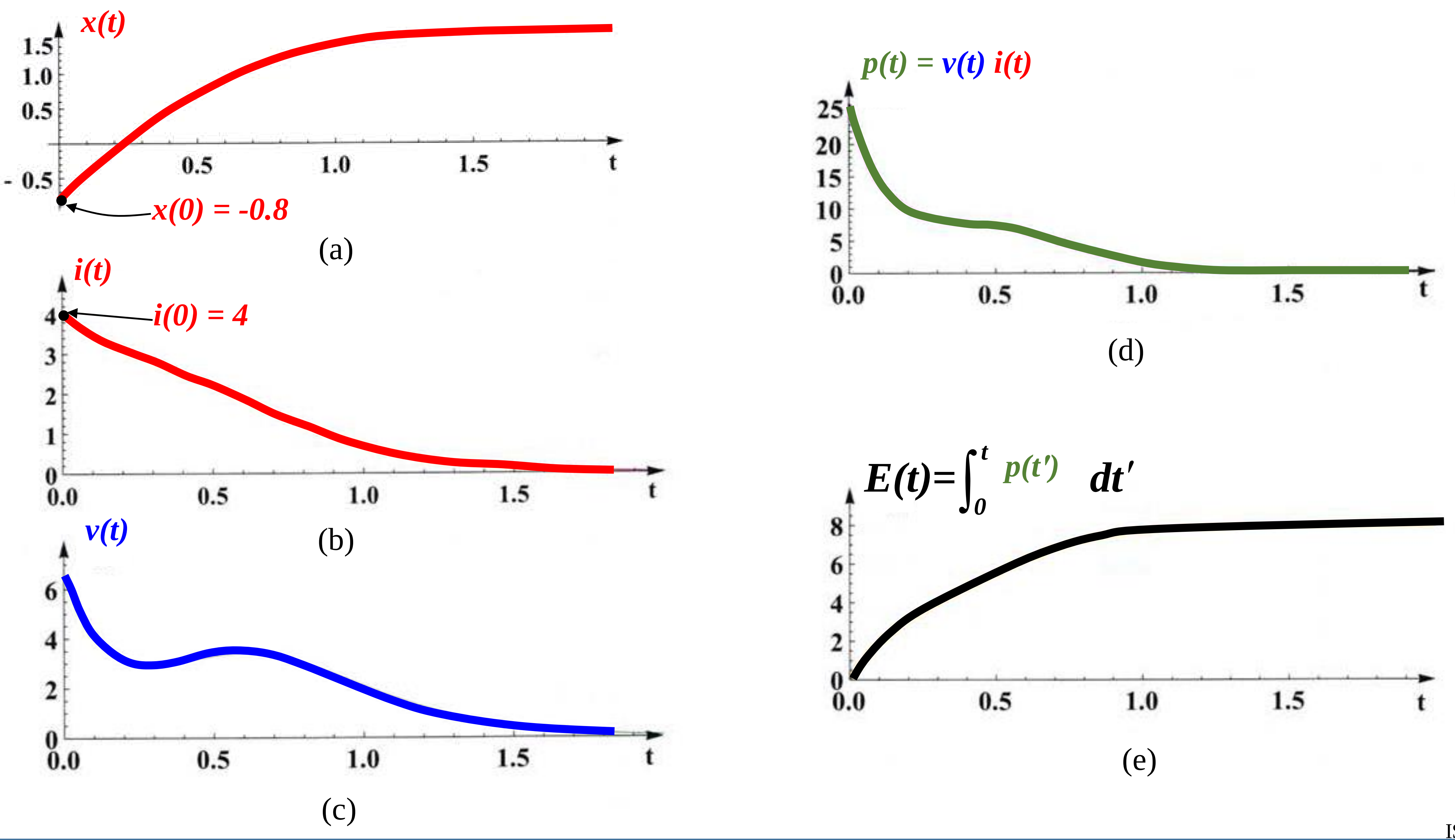
Pseudo Potential Energy H_x (x) *Pseudo Kinetic Energy H_i (i)*



The Hamiltonian has a constant value $H(x, i) = H_0$ along each curve.

Hamiltonian

$$H(x, i) = x + \frac{1}{3}x^3 + i$$


x(t)
1.5
1.0
0.5
- 0.5
0.5
1.0
1.5
t
x(0) = -0.8
(a)
i(t)
i(0) = 4
4
3
2
1
0
0.0
0.5
1.0
1.5
t
(b)
v(t)
6
4
2
0
0.0
0.5
1.0
1.5
t
(c)
p(t) = v(t) i(t)
25
20
15
10
5
0
0.0
0.5
1.0
1.5
t
(d)
$E(t)=\int_0^t p(t')\, dt'$
8
6
4
2
0
0.0
0.5
1.0
1.5
t
(e)

Pseudo Potential Energy

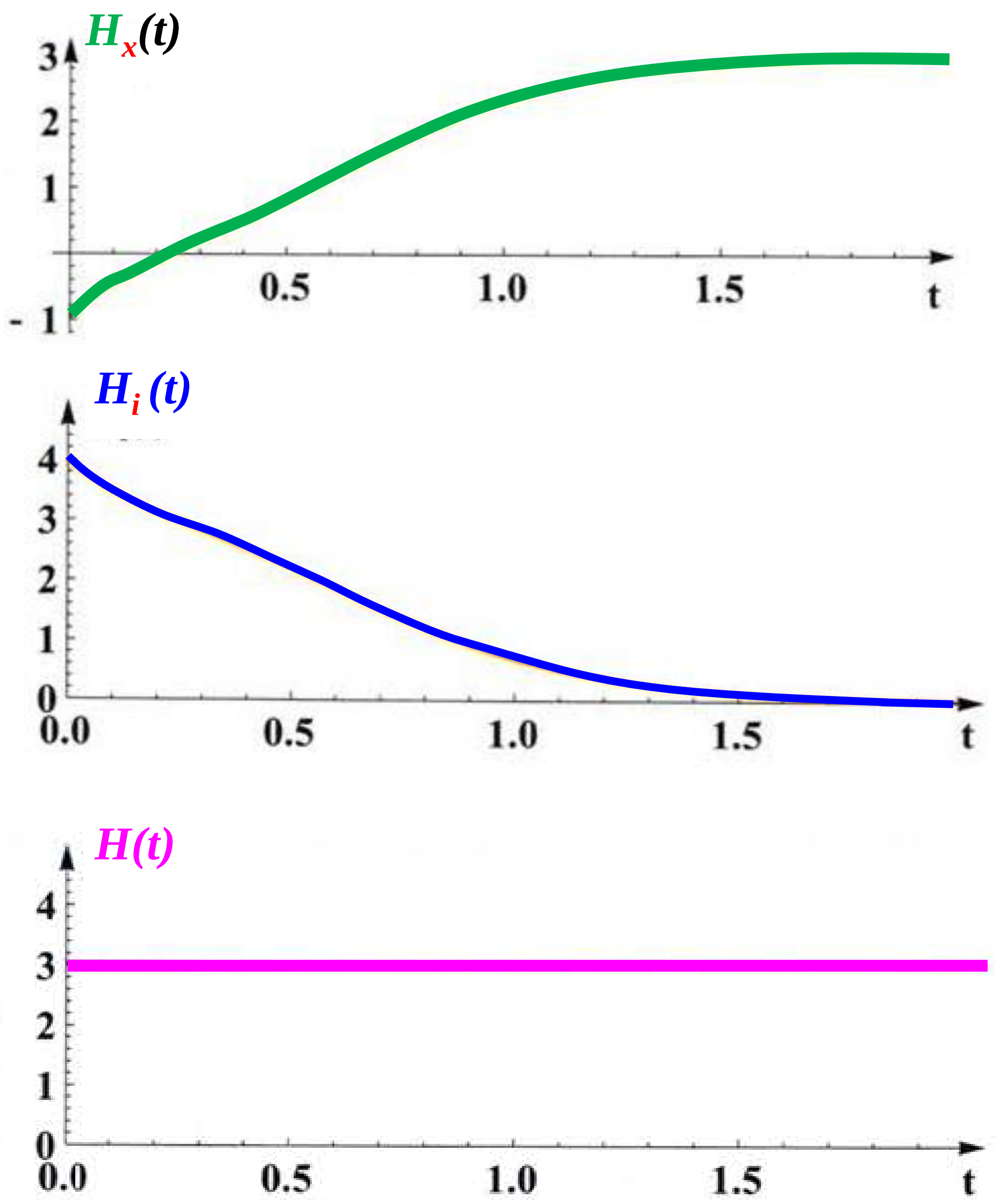
$$H_x = x + \frac{1}{3}x^3$$

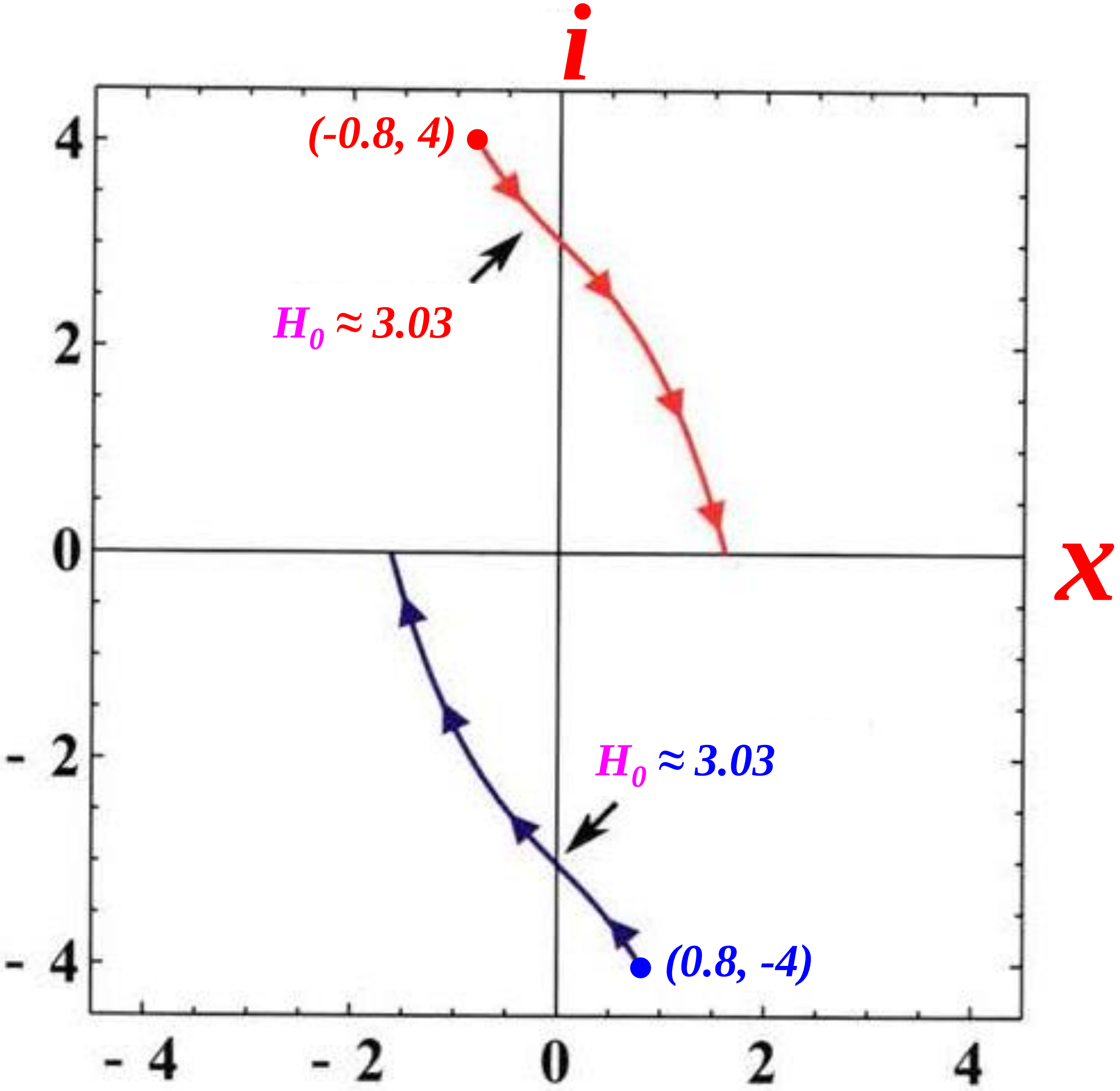
Pseudo Kinetic Energy

$$H_i = i$$

Hamiltonian

$$H = \underbrace{H_x}_{\text{Pseudo Potential Energy}} + \underbrace{H_i}_{\text{Pseudo Kinetic Energy}}$$





Two typical trajectories which tend to the x-axis along on the contour $H_0 \approx 3.03$.

Shocking Revelation !

Not all

Hamiltonian Systems

are

Conservative

Question :

What *physical* quantity is conserved in the *Hamiltonian* H(x , i) ?

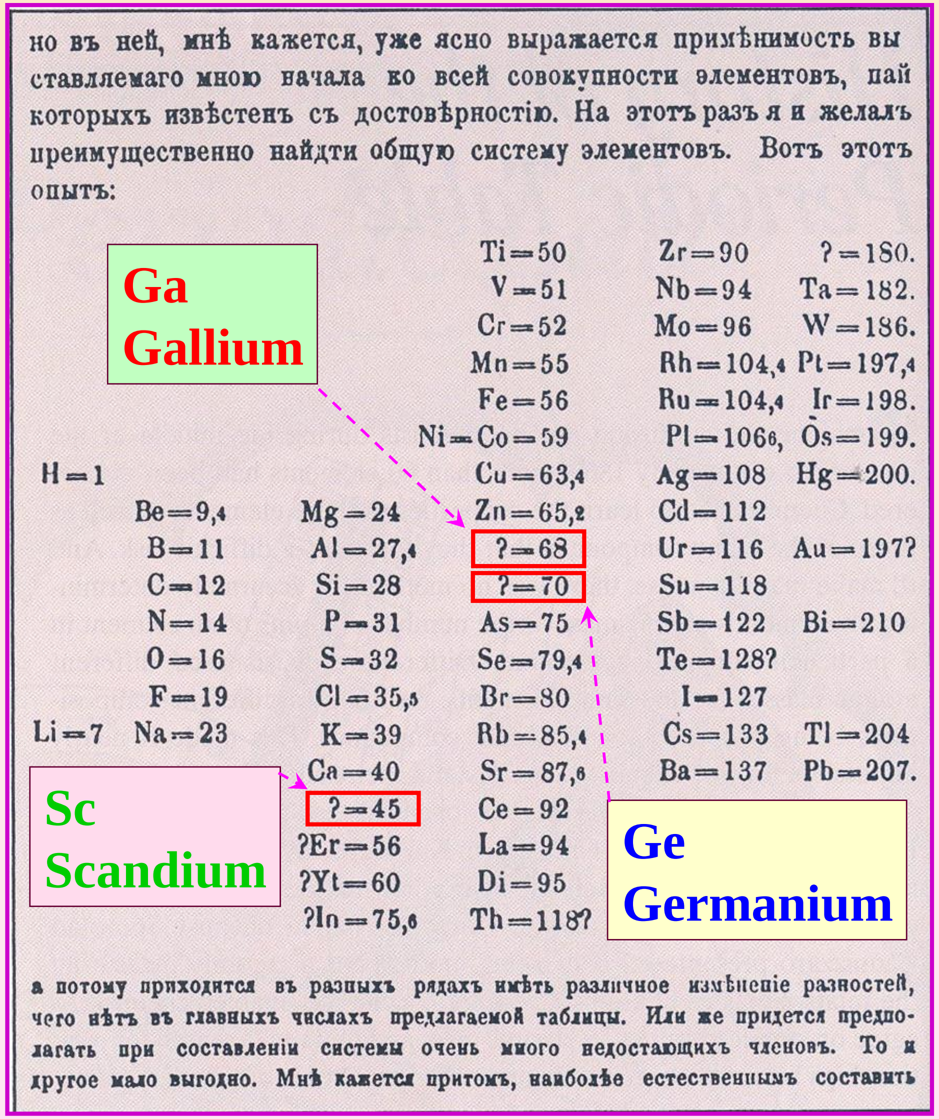
Answer :

The *Hamiltonian* H(x , i)

conserved the total *flux*

$\varphi(t)$	$=$	$\varphi(t)$	$+$	$\varphi(t)$
Total		***Inductor***		***Memristor***

Mendeleev’s First Published Periodic Table, 1869



но въ ней, мнѣ кажется, уже ясно выражается примѣнимость выставляемаго мною начала ко всей совокупности элементовъ, пай которыхъ извѣстенъ съ достовѣрностію. На этотъ разъ я и желалъ преимущественно найдти общую систему элементовъ. Вотъ этотъ опытъ:

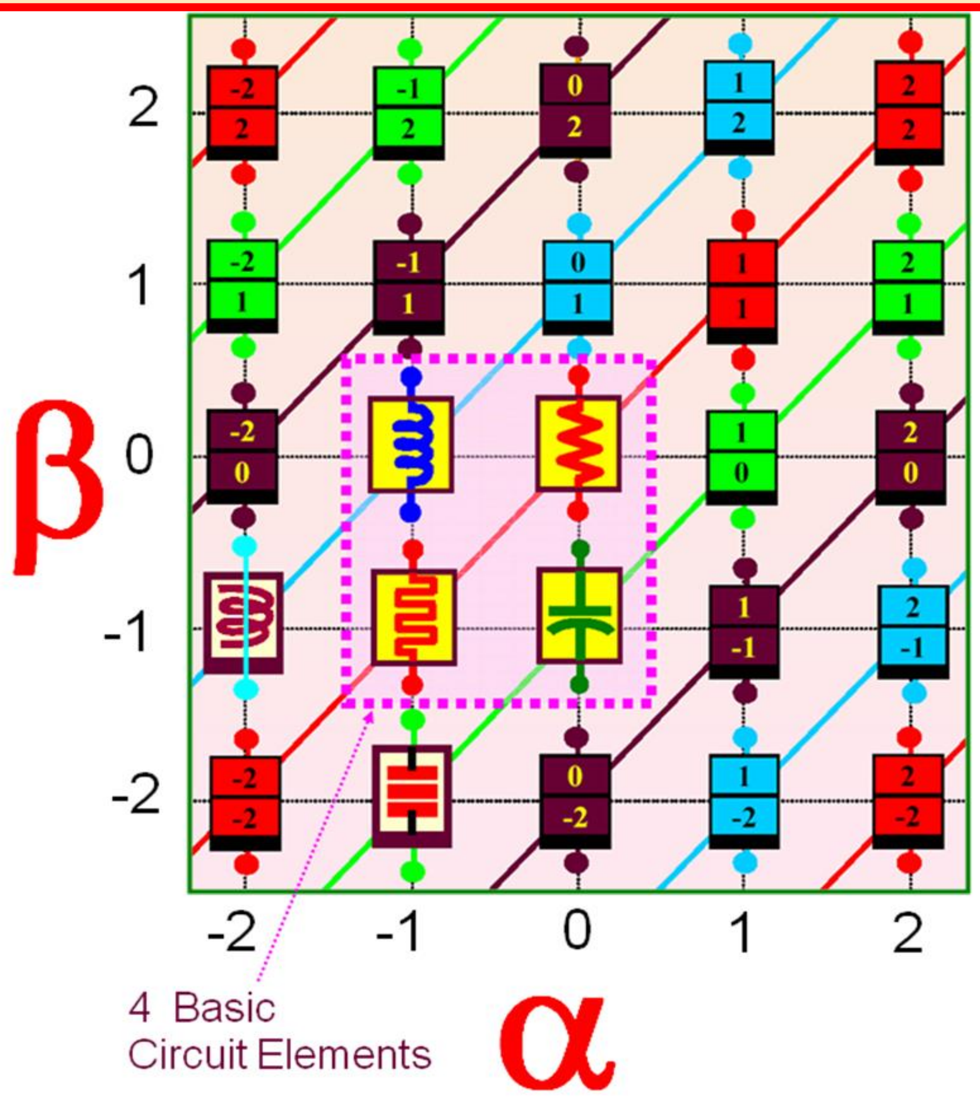
			Ti=50	Zr=90	?=180.
			V=51	Nb=94	Ta=182.
			Cr=52	Mo=96	W=186.
			Mn=55	Rh=104,4	Pt=197,4
			Fe=56	Ru=104,4	Ir=198.
			Ni=Co=59	Pl=106,6,	Os=199.
H=1			Cu=63,4	Ag=108	Hg=200.
	Be=9,4	Mg=24	Zn=65,2	Cd=112	
	B=11	Al=27,4	?=68	Ur=116	Au=197?
	C=12	Si=28	?=70	Su=118	
	N=14	P=31	As=75	Sb=122	Bi=210
	O=16	S=32	Se=79,4	Te=128?	
	F=19	Cl=35,5	Br=80	I=127	
Li=7	Na=23	K=39	Rb=85,4	Cs=133	Tl=204
		Ca=40	Sr=87,6	Ba=137	Pb=207.
		?=45	Ce=92		
		?Er=56	La=94		
		?Yt=60	Di=95		
		?In=75,6	Th=118?		

а потому приходится въ разныхъ рядахъ имѣть различное измѣненіе разностей, чего нѣтъ въ главныхъ числахъ предлагаемой таблицы. Или же придется предполагать при составленіи системы очень много недостающихъ членовъ. То и другое мало выгодно. Мнѣ кажется притомъ, наиболѣе естественнымъ составить

The First 25 Circuit Elements

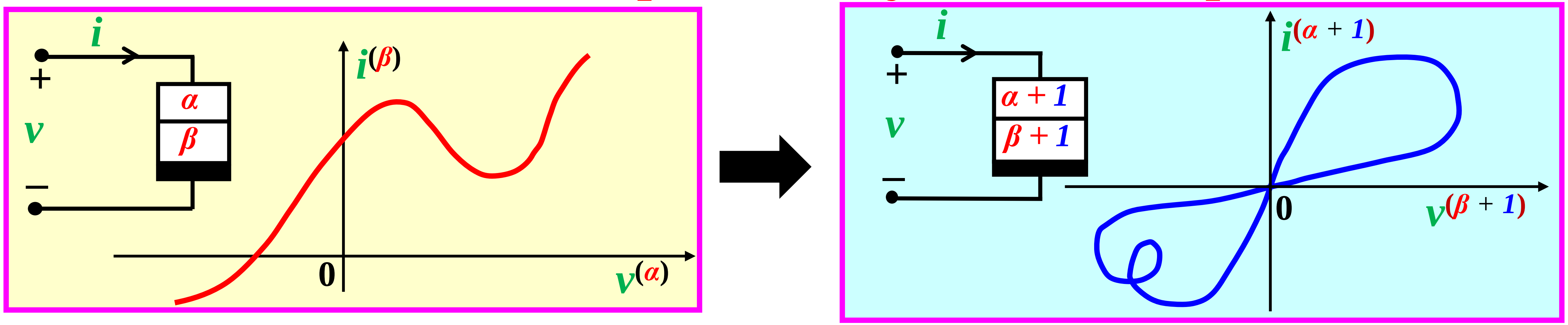
$$v^{(\alpha)}(t) \triangleq \begin{cases} \dfrac{d^{\alpha} v(t)}{dt^{\alpha}}, & \text{if } \alpha = 1, 2, \ldots \infty \\ v(t), & \text{if } \alpha = 0 \\ \int_{-\infty}^{t} v(\tau)\, d\tau, & \text{if } \alpha = -1 \\ \int_{-\infty}^{t} \int_{-\infty}^{\tau_{|\alpha|}} \cdots \int_{-\infty}^{\tau_2} v(\tau_1)\, d\tau_1 d\tau_2 \cdots d\tau_{|\alpha|}, & \text{if } \alpha = -2, -3, \ldots \infty \end{cases}$$

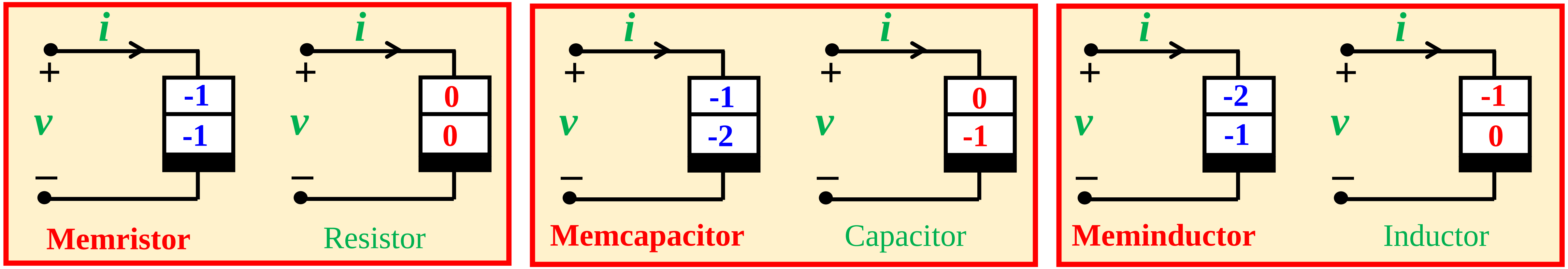
$$i^{(\beta)}(t) \triangleq \begin{cases} \dfrac{d^{\beta} i(t)}{dt^{\beta}}, & \text{if } \beta = 1, 2, \ldots \infty \\ i(t), & \text{if } \beta = 0 \\ \int_{-\infty}^{t} i(\tau)\, d\tau, & \text{if } \beta = -1 \\ \int_{-\infty}^{t} \int_{-\infty}^{\tau_{|\beta|}} \cdots \int_{-\infty}^{\tau_2} i(\tau_1)\, d\tau_1 d\tau_2 \cdots d\tau_{|\beta|}, & \text{if } \beta = -2, -3, \ldots \infty \end{cases}$$



Universal Pinched Hysteresis Loop Theorem

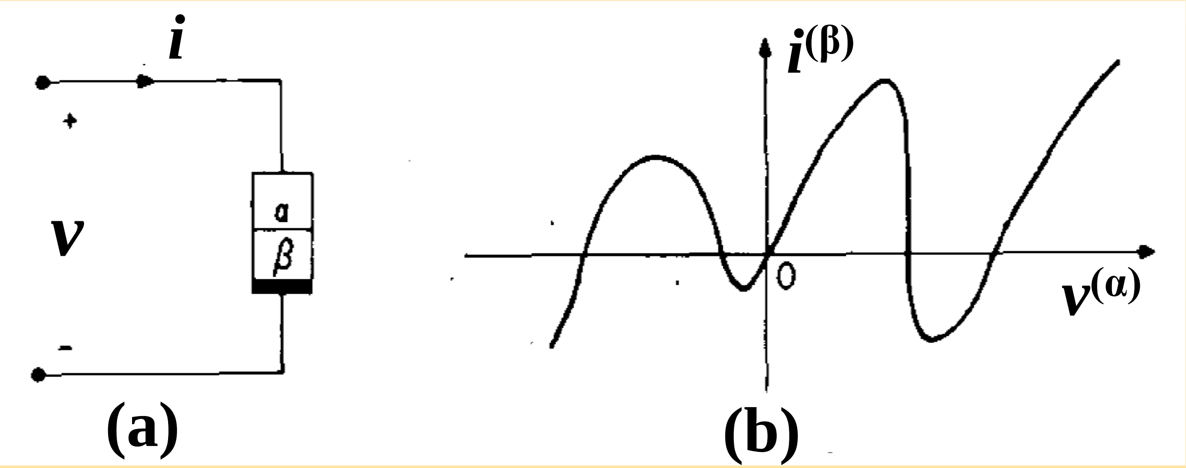
The $((\alpha + 1), (\beta + 1))$ element associated with any (α, β) element *must* exhibit a *pinched hysteresis loop*.



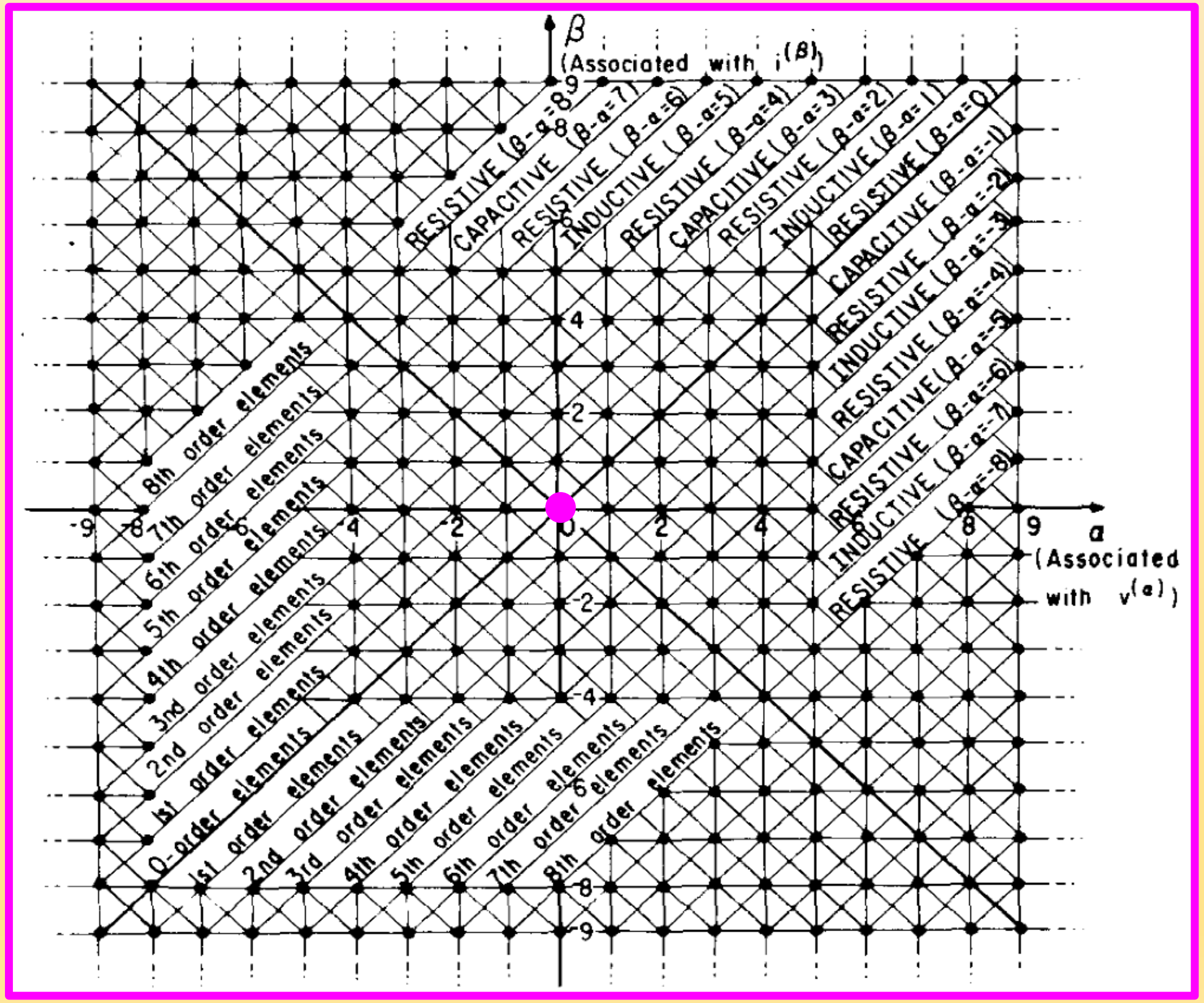


Example 1 Example 2 Example 3

$(\alpha\ ,\ \beta)$ - *Elements*



(a) *Symbol for a* $v^{(\alpha)} - i^{(\beta)}$ *element.* (b) *The constitutive relation of a curve or subset of points in the* $v^{(\alpha)} - i^{(\beta)}$ *plane.*



Circuit-element array: Each dot with coordinates (α, β) *denotes a* $v^{(\alpha)} - i^{(\beta)}$ *element.*

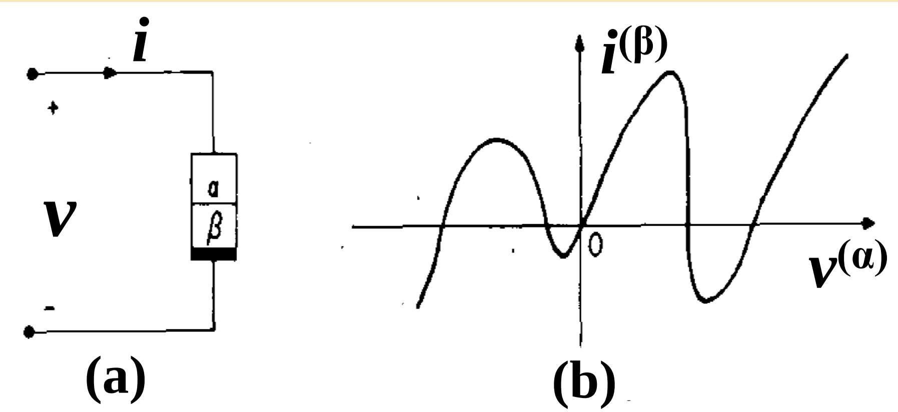
Leon Chua

Device Modelling Via Basic Nonlinear Circuit Elements

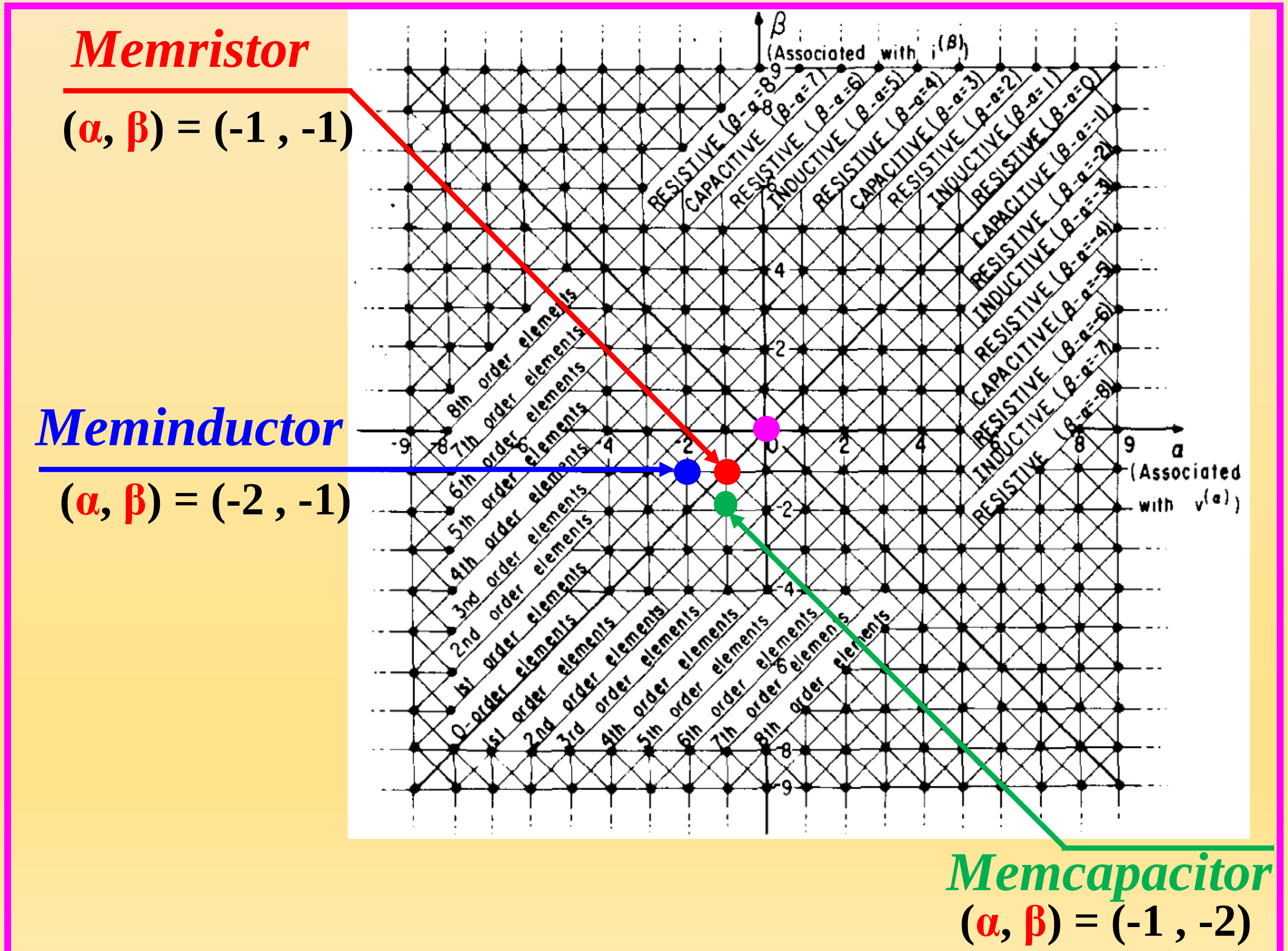
IEEE Transactions on Circuits and Systems vol. CAS-27, No. 11, pp. 1014-1044,

1980

(α , β) - Elements



(a) *Symbol for a* $v^{(\alpha)} - i^{(\beta)}$ *element.* (b) *The constitutive relation of a curve or subset of points in the* $v^{(\alpha)} - i^{(\beta)}$ *plane.*

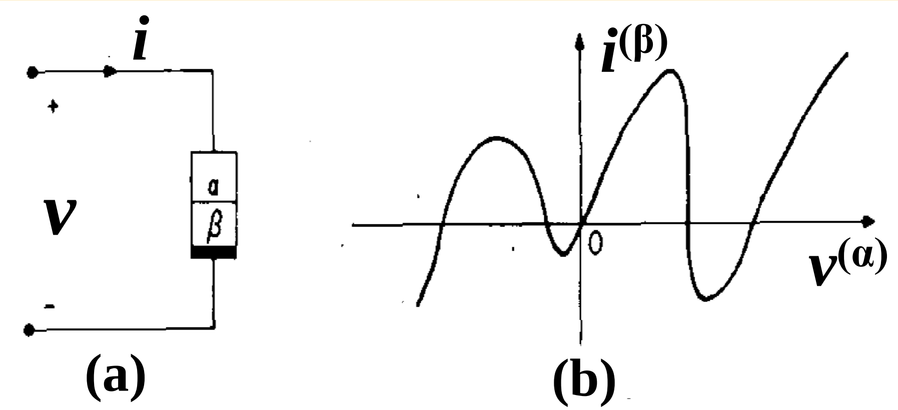


Leon Chua

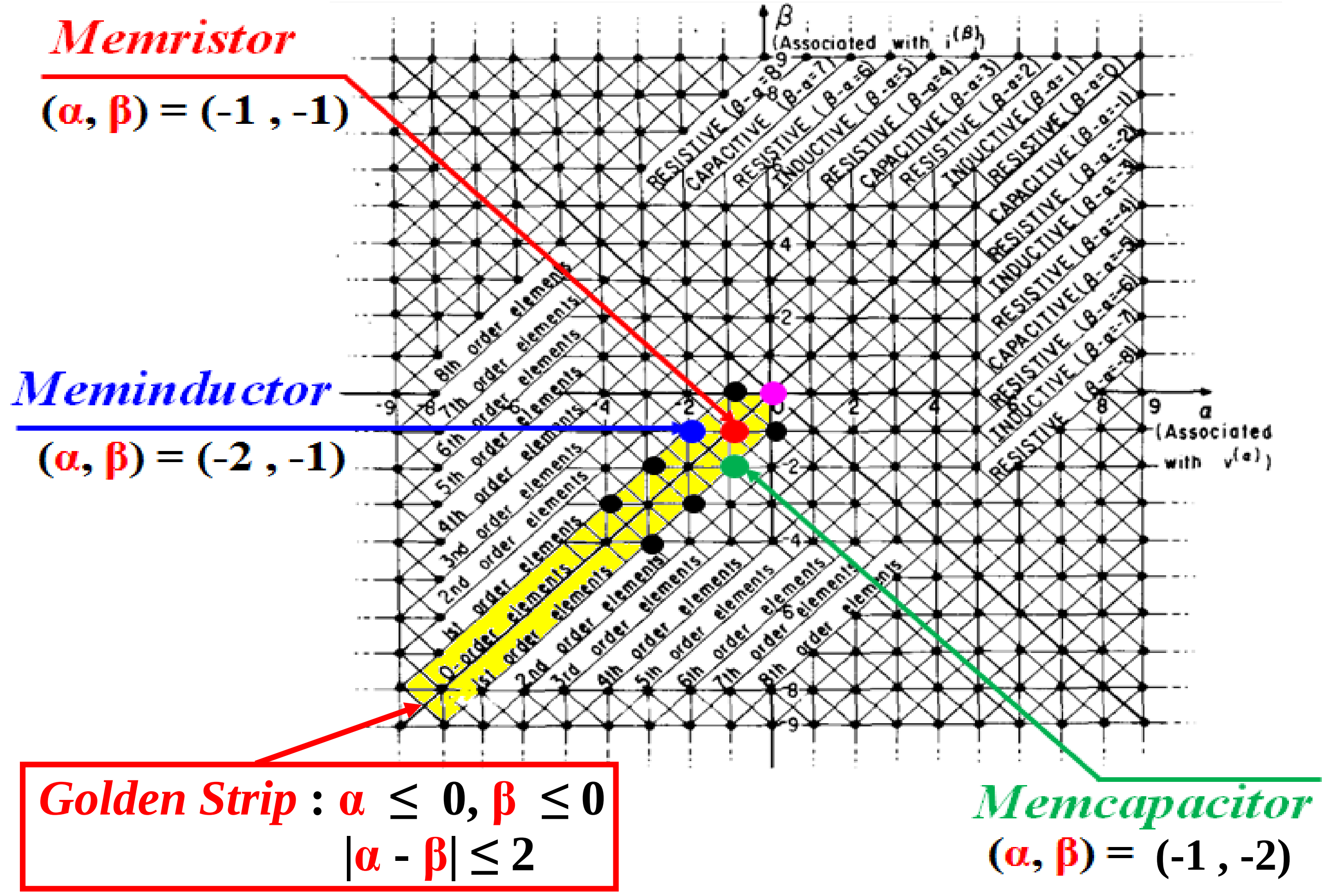
Device Modelling Via Basic Nonlinear Circuit Elements

IEEE Transactions on Circuits and Systems
vol. CAS-27, No. 11, pp. 1014-1044,
1980

The Golden Strip



(a) *Symbol for a* $v^{(\alpha)} - i^{(\beta)}$ *element.* **(b)** *The constitutive relation of a curve or subset of points in the* $v^{(\alpha)} - i^{(\beta)}$ *plane.*



Leon Chua

Device Modelling Via Basic Nonlinear Circuit Elements

IEEE Transactions on Circuits and Systems
vol. CAS-27, No. 11, pp. 1014-1044,
1980

Theorem: *Only elements belonging to the Golden Strip are passive and physically realizable.*

CIRCUIT THEORY AND APPLICATIONS, VOL. 11, 187–206 (1983)

HIGH-ORDER NON-LINEAR CIRCUIT ELEMENTS: CIRCUIT-THEORETIC PROPERTIES

LEON O. CHUA AND ELLEN W. SZETO

Department of Electrical Engineering and Computer Sciences, and Electronics Research Laboratory, University of California, Berkeley, CA 94720, U.S.A.

SUMMARY

Higher- and mixed-order non-linear circuit elements have been introduced to provide a logically complete formulation for non-linear circuit theory. In this paper, we analyse the circuit-theoretic properties of these elements, including *reciprocity*, *passivity* and *losslessness*. We have derived necessary and sufficient conditions for a higher- or mixed-order *n-port* element to be reciprocal or antireciprocal. We have shown that under very mild assumptions, most *non-linear* higher-order *2-terminal* elements are active and not lossless. Finally, we show that the number of lossless *linear* higher-order 2-terminal elements far exceeds that of the passive linear elements.

Inspiration from Chemistry

The latest *Periodic Table* contains

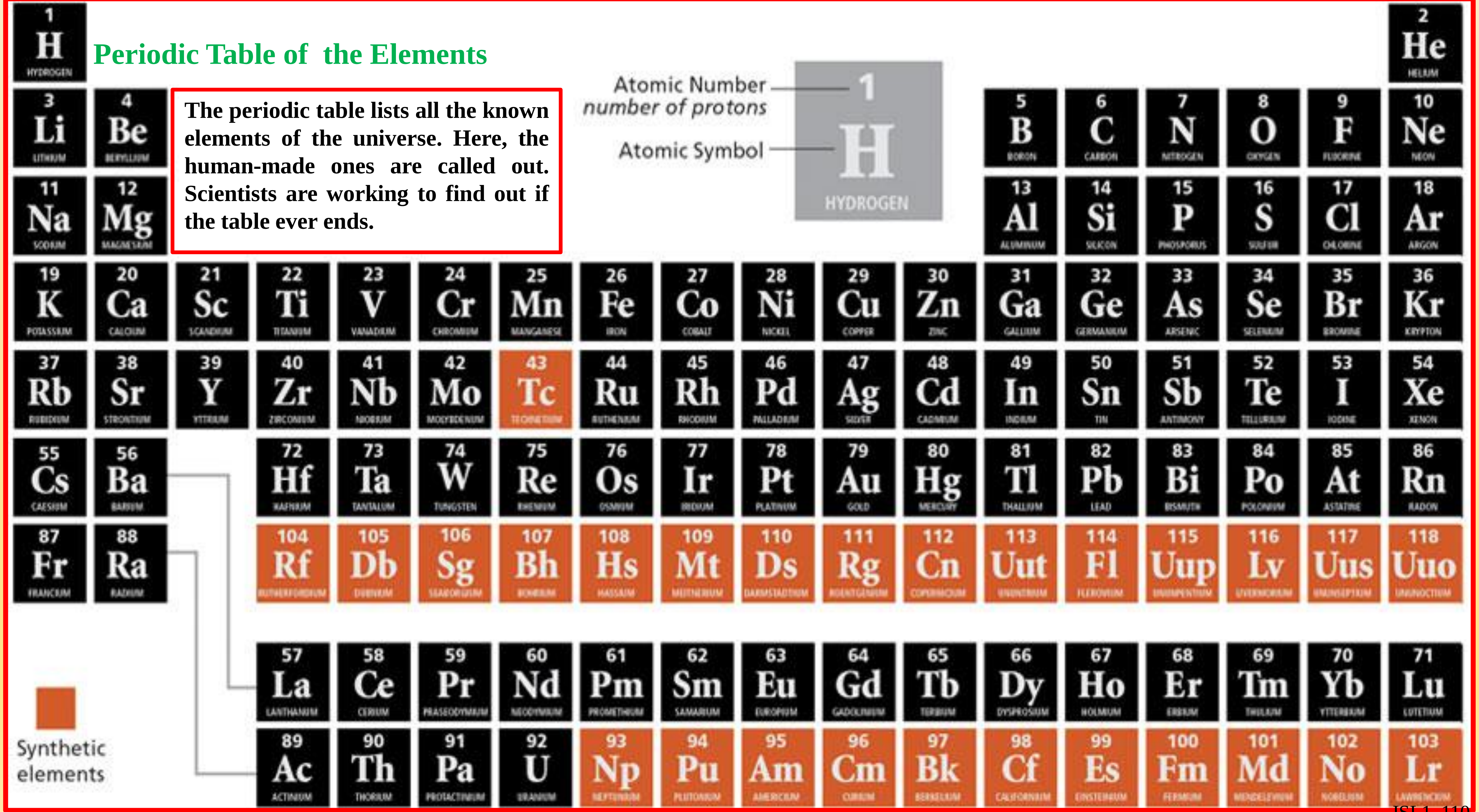
***118 elements* :**

- ***92 elements* exist in *nature*.**
- ***26 elements* are *artificially synthesized*.**

Most *synthesized elements* are *unstable* --- some exist for only a few *milliseconds*, then decomposed into two lighter elements.

Periodic Table of the Elements

The periodic table lists all the known elements of the universe. Here, the human-made ones are called out. Scientists are working to find out if the table ever ends.



Nonlinear Circuit Theory* predicts the existence of *infinitely many passive* (α , β) - *elements

So far, *only 4 passive circuit elements* have been built *without power supply*:

- ***Resistor***
- ***Inductor***
- ***Capacitor***
- ***Memristor***

A Cap for Number of Chemical Elements

No Element beyond

137

can exist ; because it would violate the laws of physics.

The Mysterious 137

Fy^{137}

Feynmanium

Fractional - Order

mem – (α , β)

Circuit elements

$$0 \leq \alpha \leq 1$$

$$0 \leq \beta \leq 1$$

Fractional Derivative

Definition : Fractional Derivative (Riemann – Liouville definition)

$$ {}_a^R D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t (t-\tau)^{n-\alpha-1} f(\tau) d\tau $$

$$ = \frac{d^n}{dt^n}\left({}_a j_t^{\,n-\alpha} f(t) \right), \quad t > a, $$

$$ n-1 \le \alpha < n, $$

where **Γ** is the gamma function and ${}_a j_t^{\beta}$ is the ***Riemann-Liouville*** integral operator defined by:

$$ {}_a j_t^{\beta} = \frac{1}{\Gamma(\beta)} \int_a^t (t-\tau)^{\beta-1} f(\tau) d\tau. $$

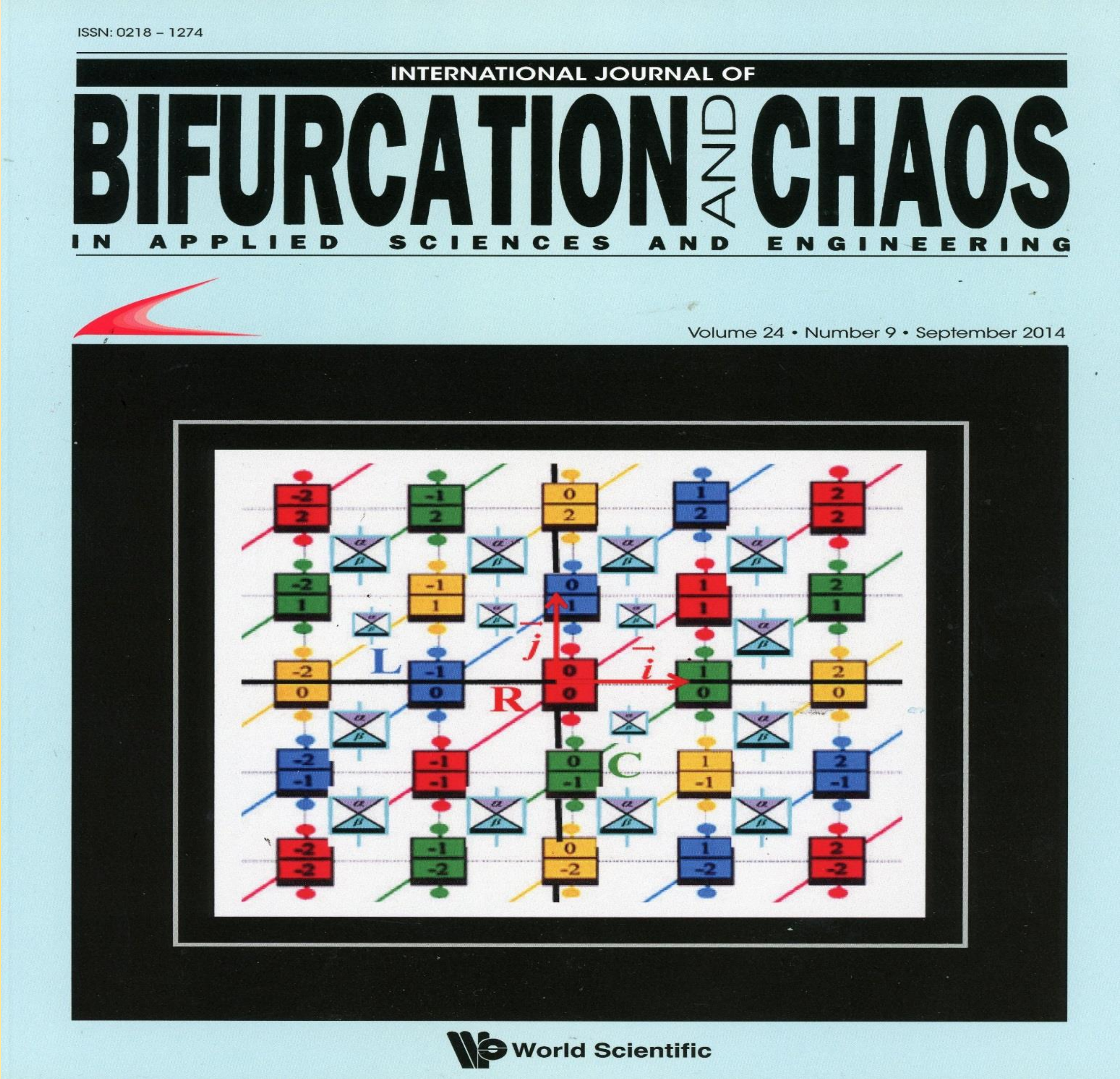
Laplace Transform of Riemann - Liouville differential Operator

The Laplace Transform of the α-order ***Riemann-Liouville differential operator*** is:

$$ L\left\{ {}_0^R D_t^{\alpha} f(t) \right\} = s^{\alpha} L\{ f(t) \} - \sum_{k=0}^{n-1} s^k \left[{}_0^R D_t^{\alpha-1-k} f(t) \right]_{t=0}. $$

For zero initial conditions we have

$$ L\left\{ {}_0 D_t^{\alpha} f(t) \right\} = s^{\alpha} L\{ f(t) \} $$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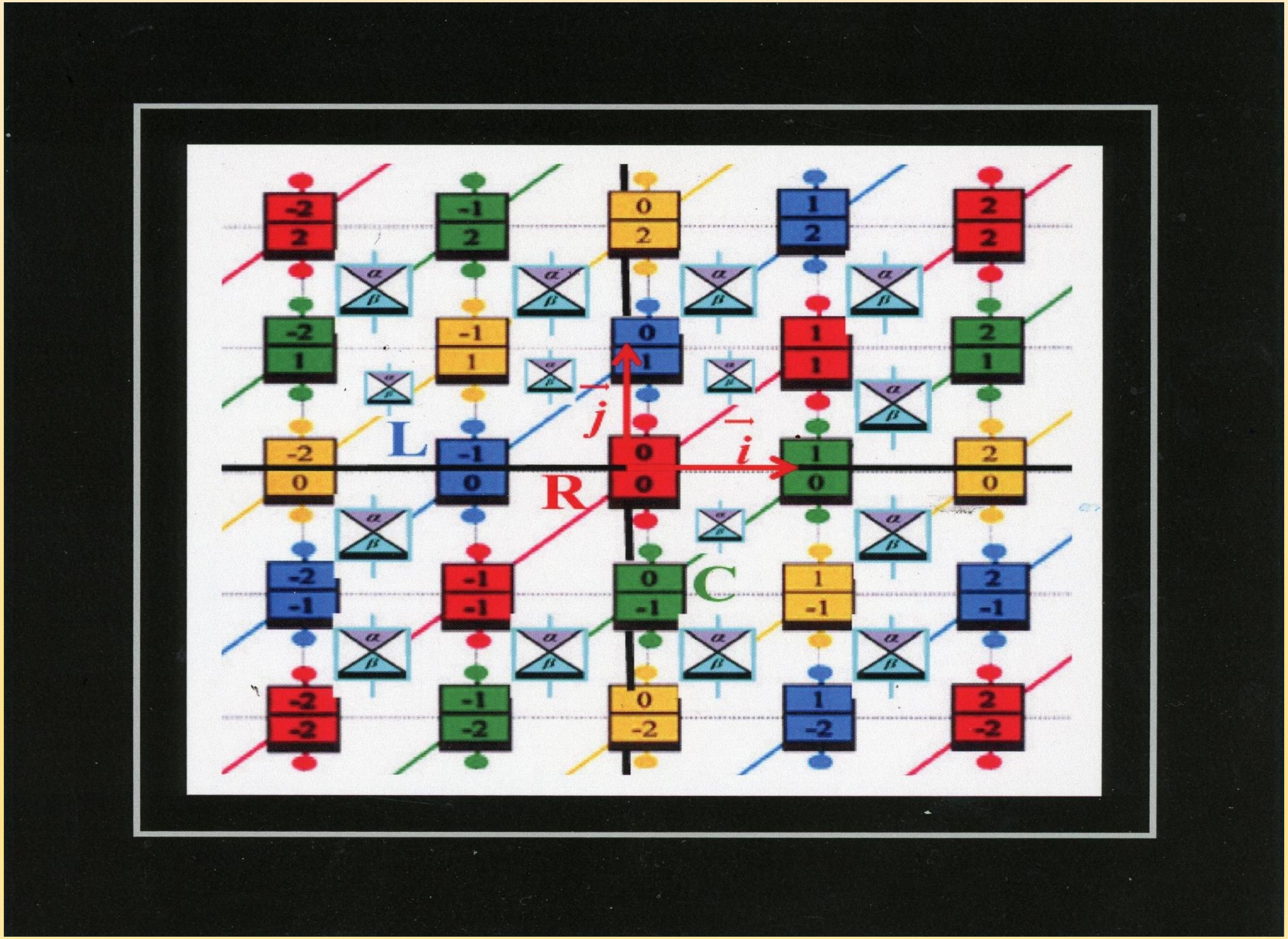
INTERNATIONAL JOURNAL OF

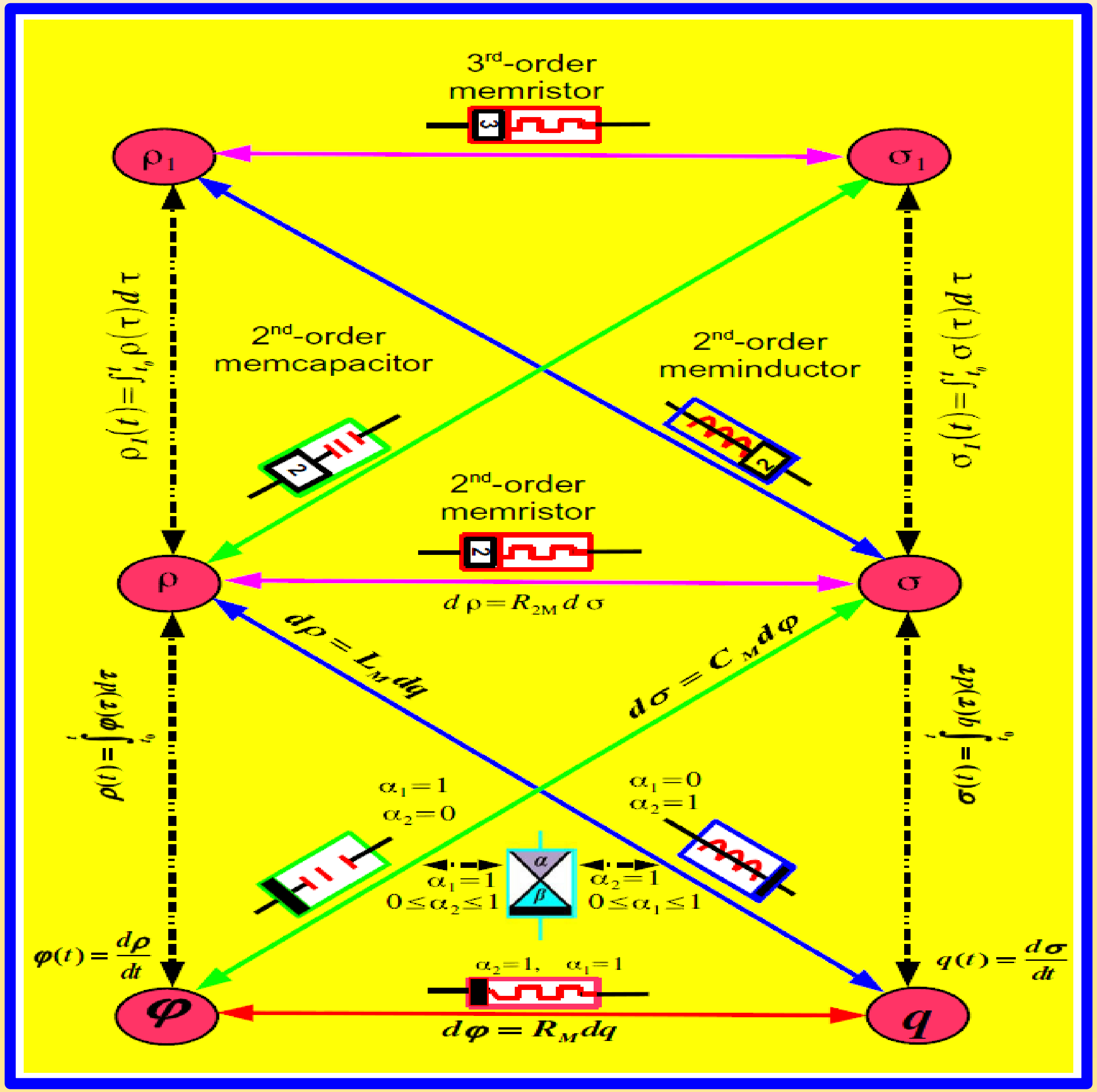
BIFURCATION AND CHAOS

IN APPLIED SCIENCES AND ENGINEERING

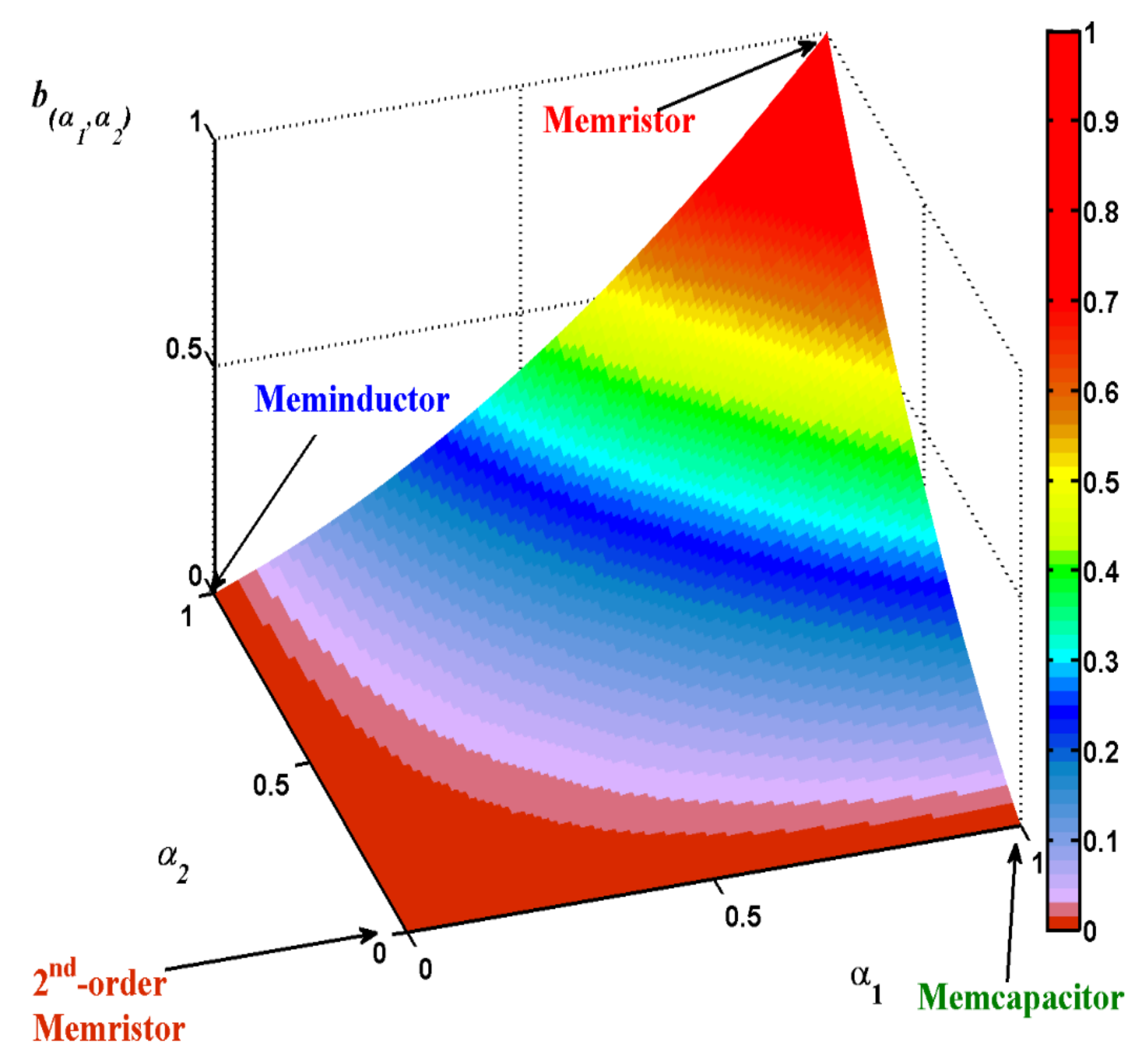
Volume 24 • Number 9 • September 2014

World Scientific


L
R
C
j
i

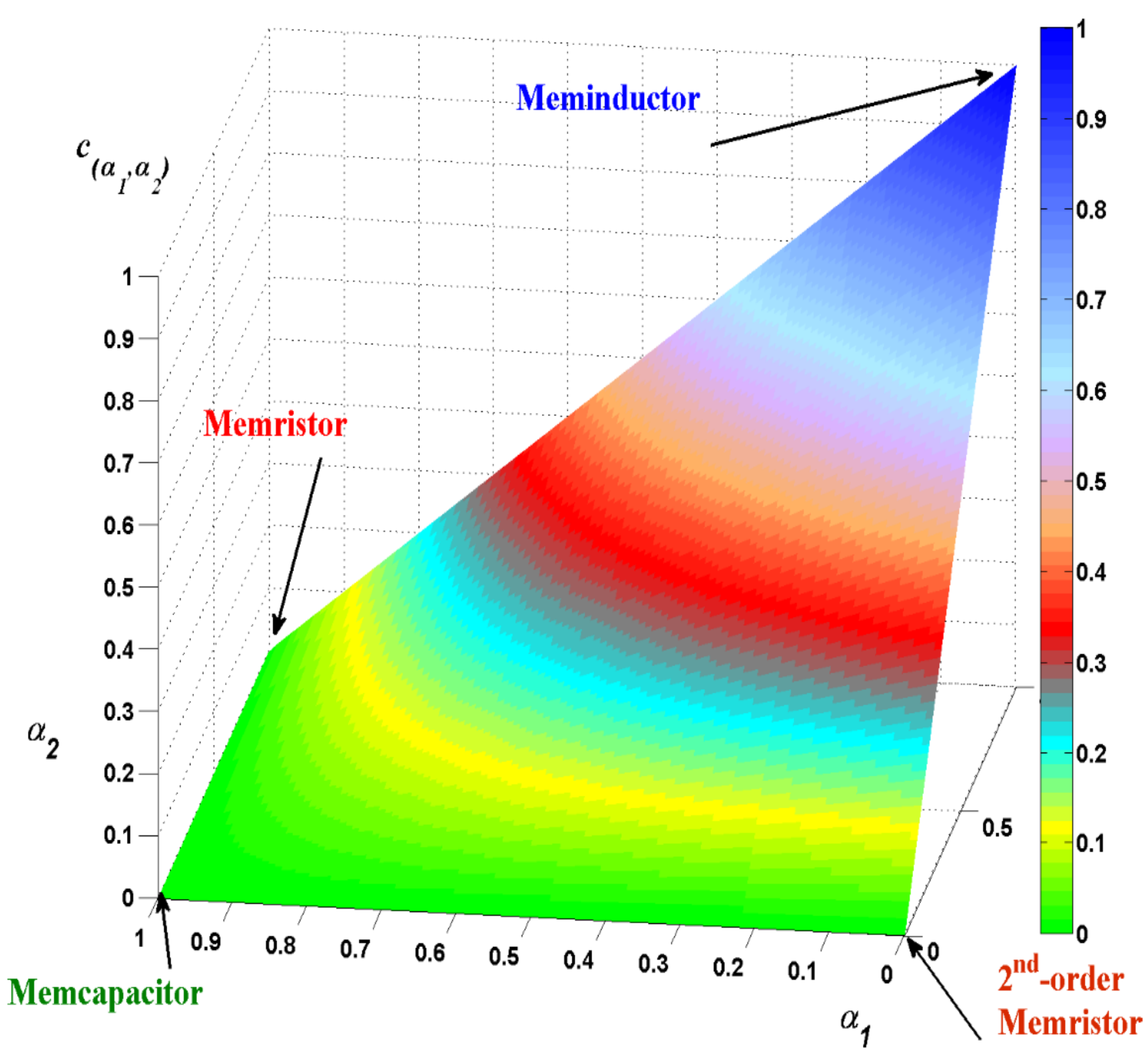


Interpolated characteristics of memfractor between a memcapacitor, a memristor, a meminductor and a second-order memristor.



Graphical representation of the coefficient

$$b_{(\alpha_1\,,\,\alpha_2)} = \alpha_1 \alpha_2 \frac{(\alpha_1 + \alpha_2)}{2}.$$



Graphical representation of the coefficient

$$c_{(\alpha_1\,,\,\alpha_2)} = \alpha_2 (1 - \alpha_1).$$

International Journal of Bifurcation and Chaos, Vol. 24, No. 9 (2014) 1430023 (29 pages)

DOI: 10.1142/S0218127414300237

Memfractance: A Mathematical Paradigm for Circuit Elements with Memory

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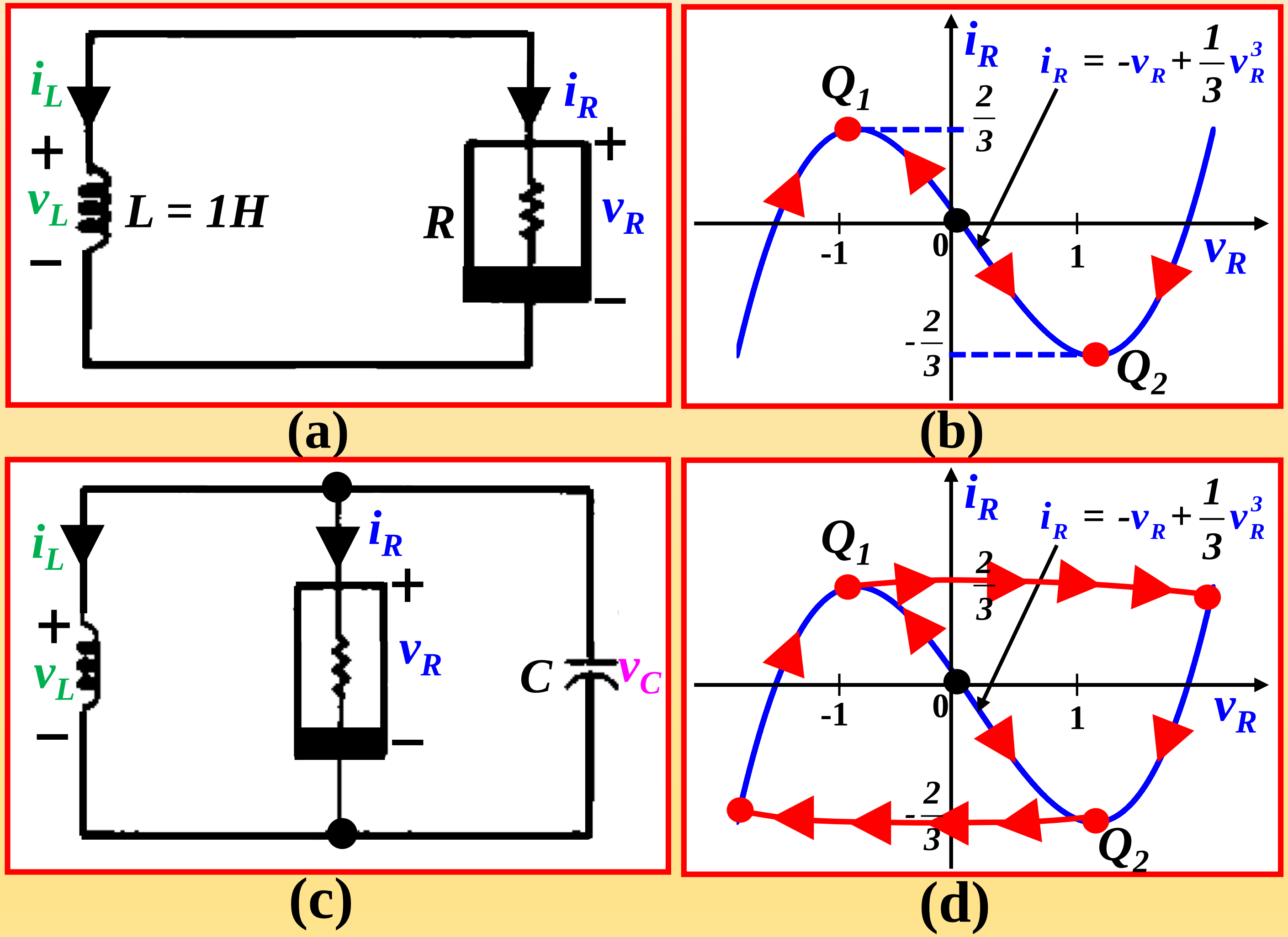
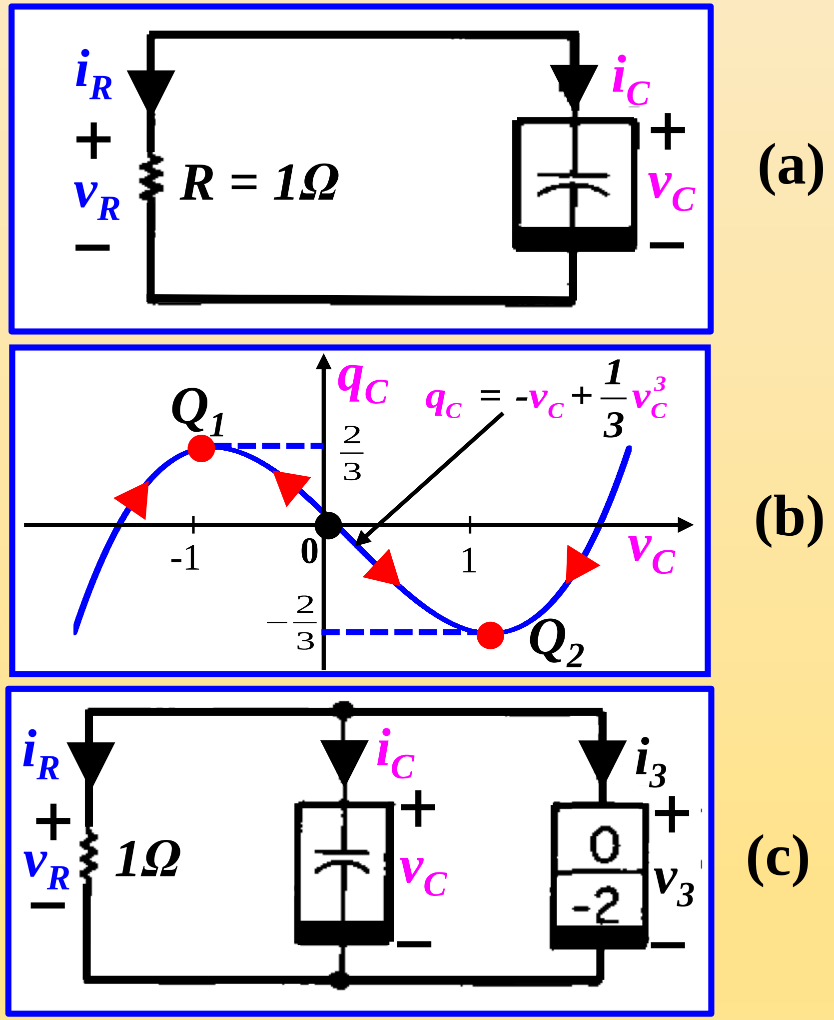
The Curse of Impasse Points



Circuit for demonstrating the necessity of connecting a capacitor across a nonmonotonic voltage-controlled Resistor.

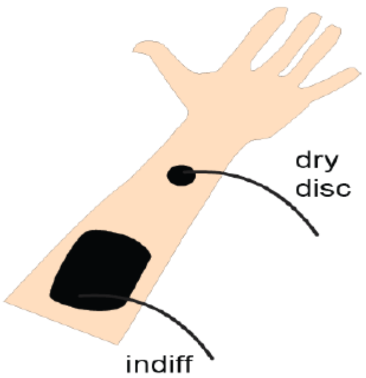


Circuit for demonstrating the necessity of connecting a $v^{(0)} - i^{(-2)}$ element across a nonmonotonic voltage-controlled Capacitor.

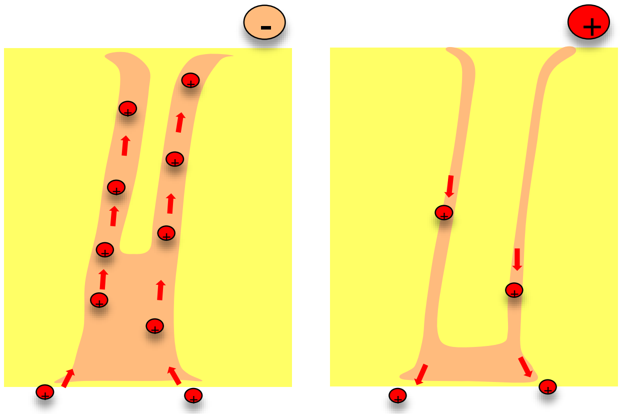
Memristors
are
Ubiquitous !

Sweat Ducts are *Memristors*

(a)

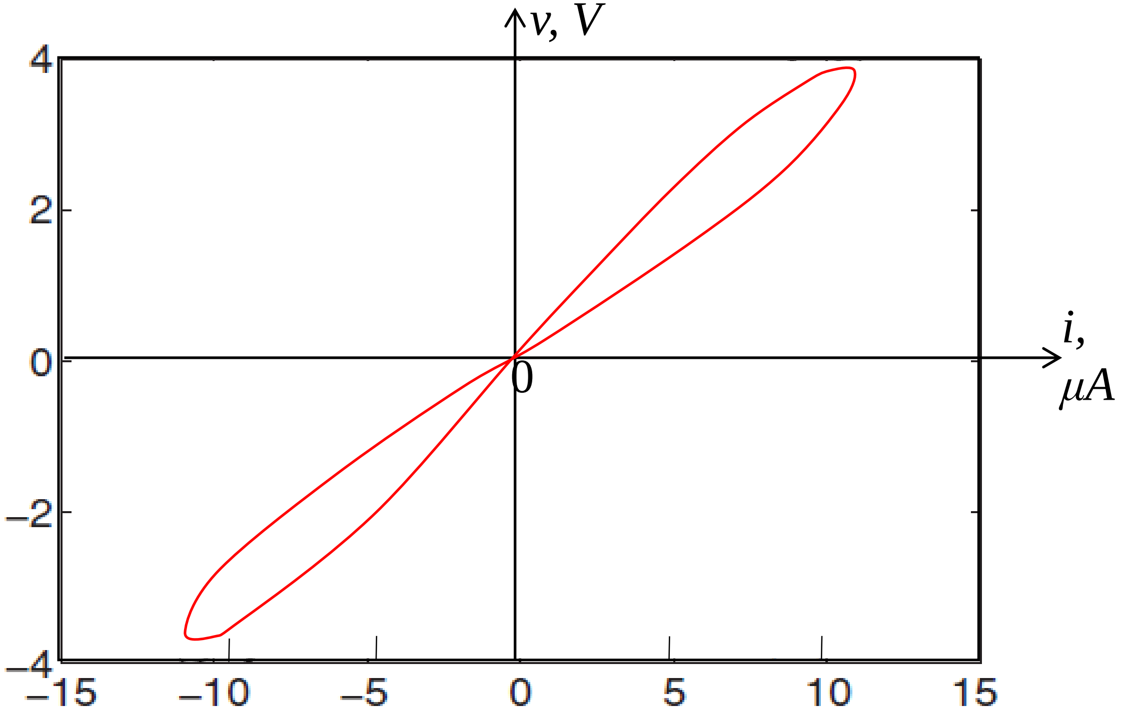


(b)

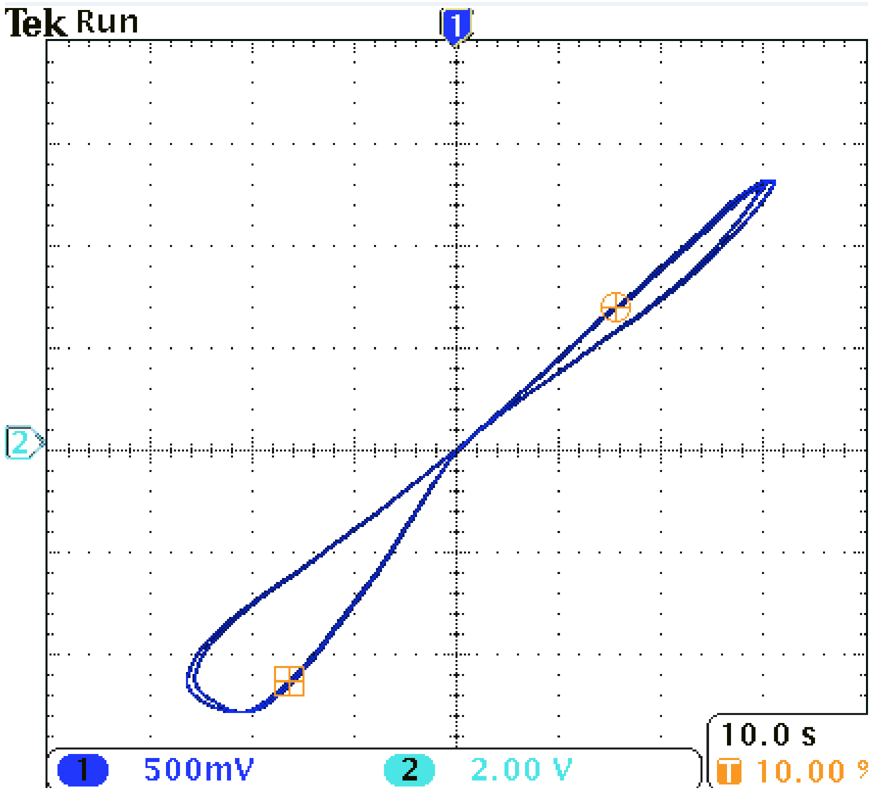


(c)

Pinched Hysteresis Loop* Measured from the *Sweat duct



Memristive Properties of Skin



Measured by:
Olivier Pabst, PhD Student

Supervisor:
Professor Orjan Martinsen

Department of Physics,
University of Oslo
September 21, 2015

Voltage over voltage plot over 2 periods. $V_{pp} = 10{:}5V$, $f = 20mHz$.
Left Hand in saline solution, other electrode on left forearm

Our body

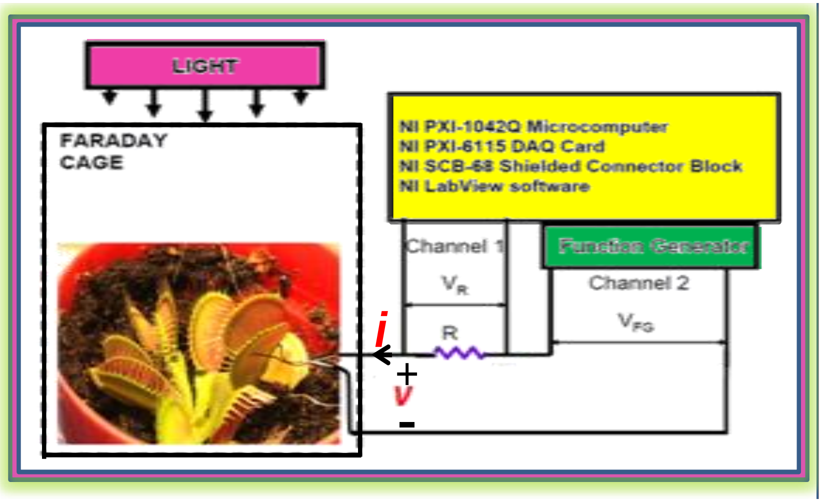
is

covered with

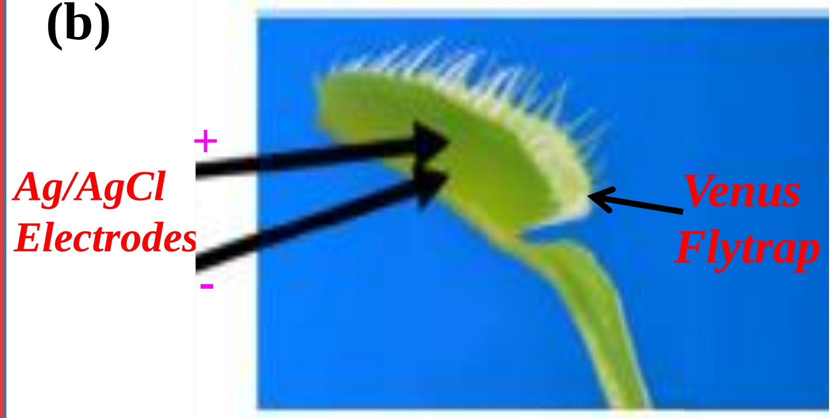
Memristors !

Pinched Hysteresis Loop *Measured from the* *Venus Flytrap*

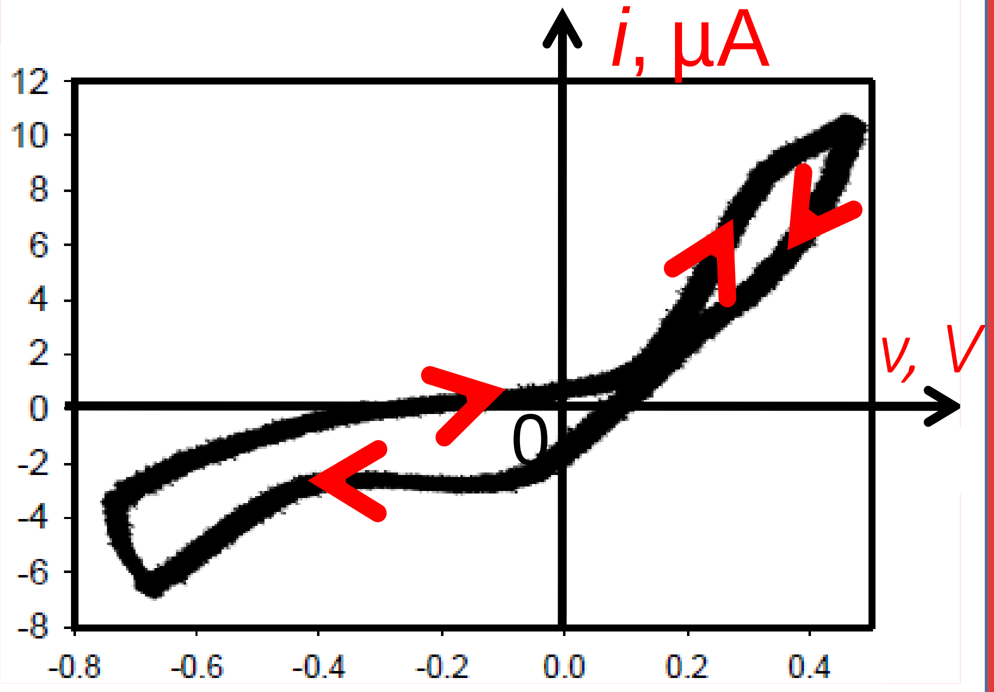
(a)



(b)

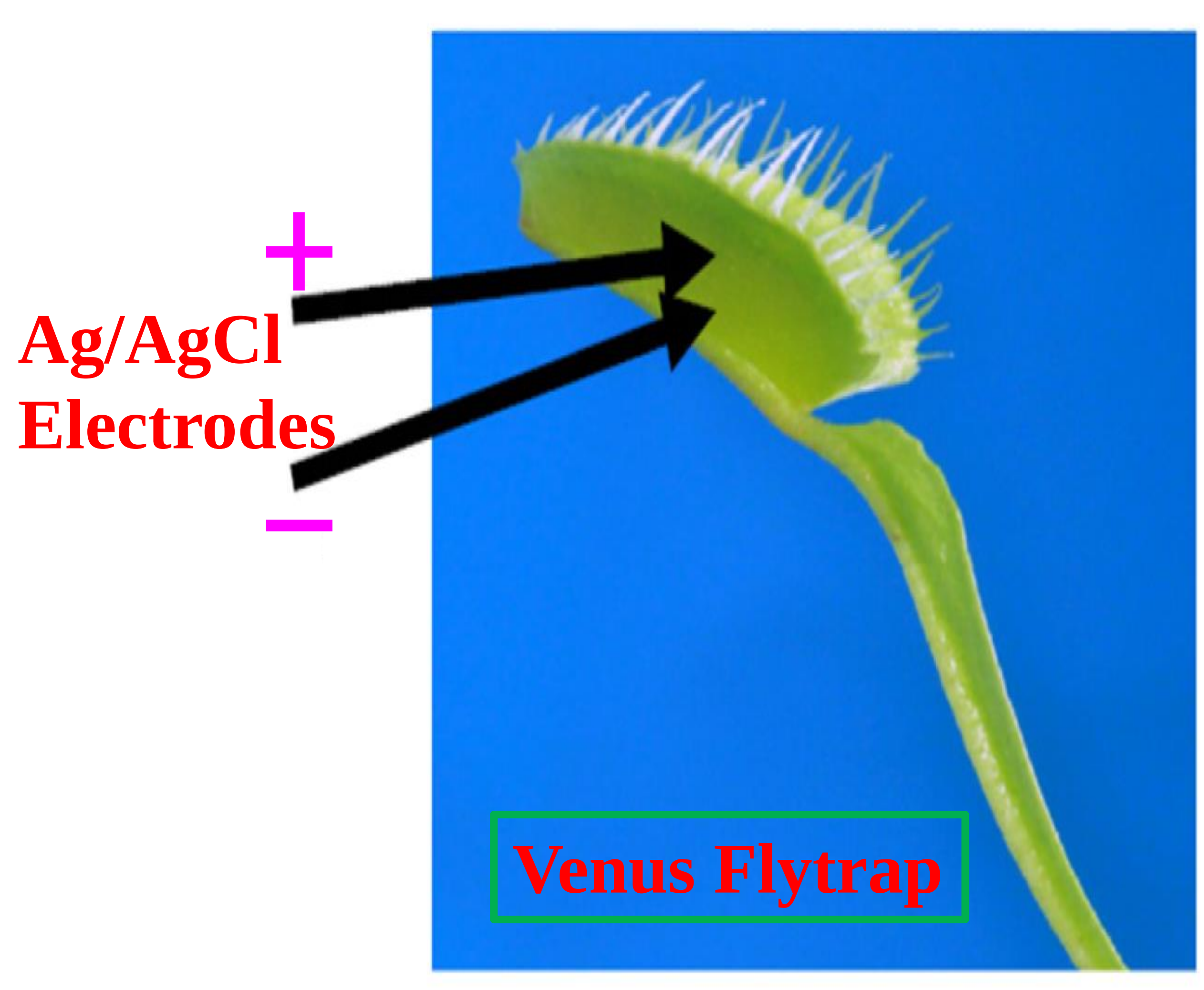


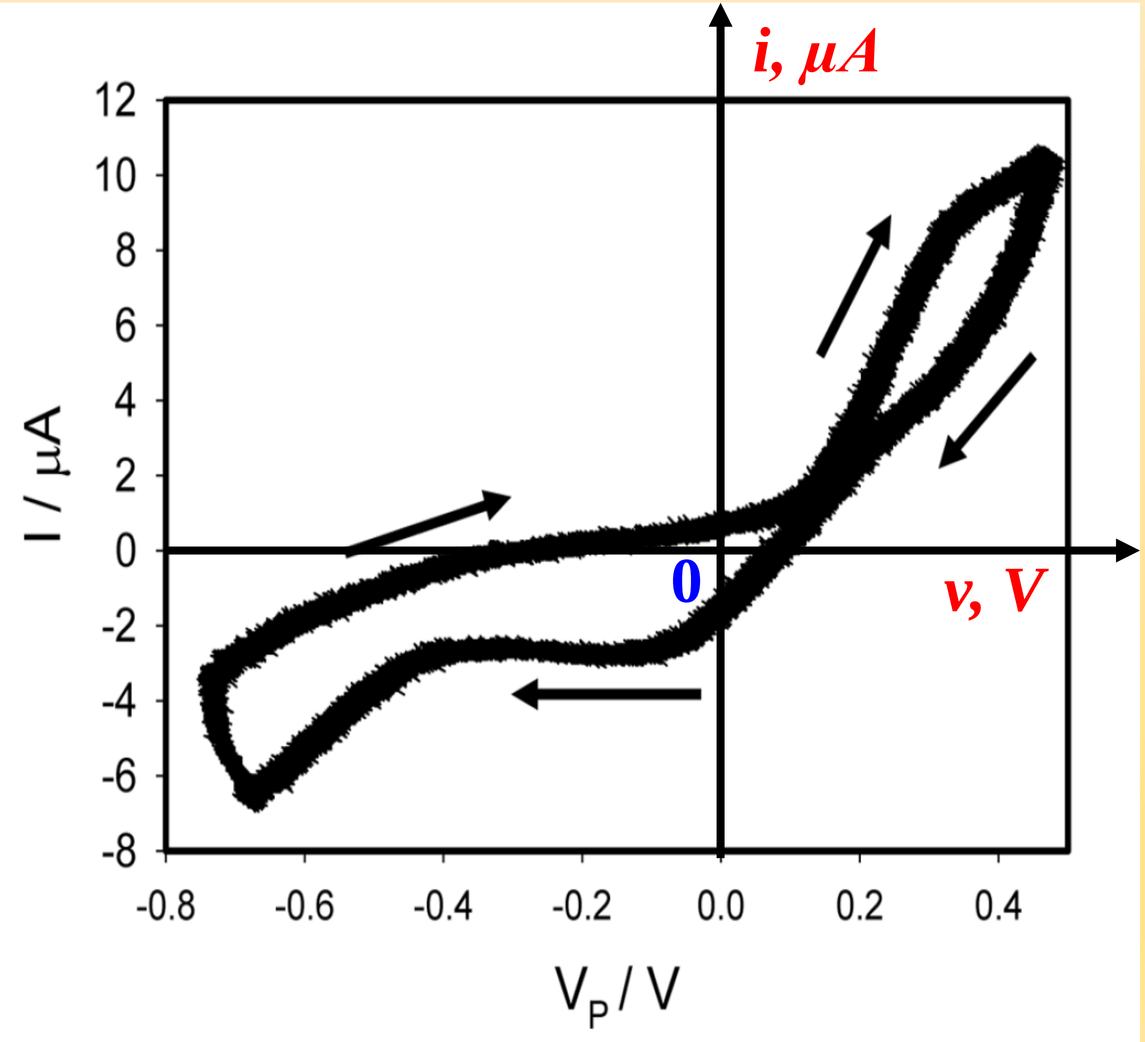
(c)



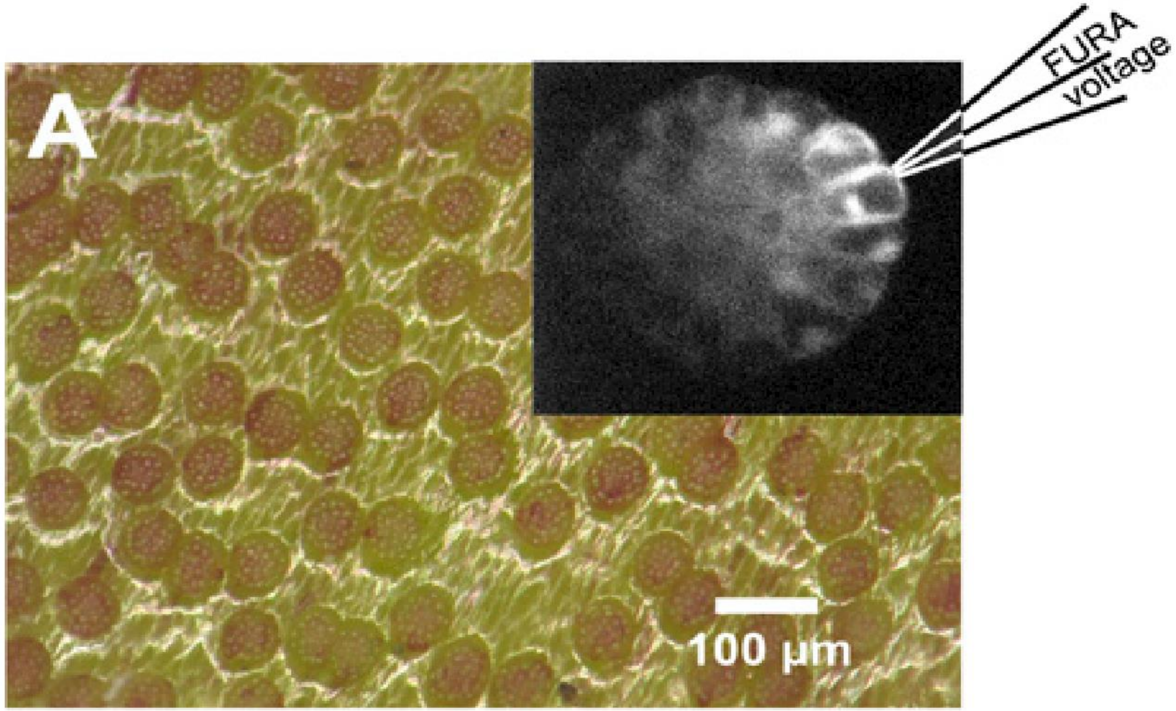
Pinched Hysteresis Loop

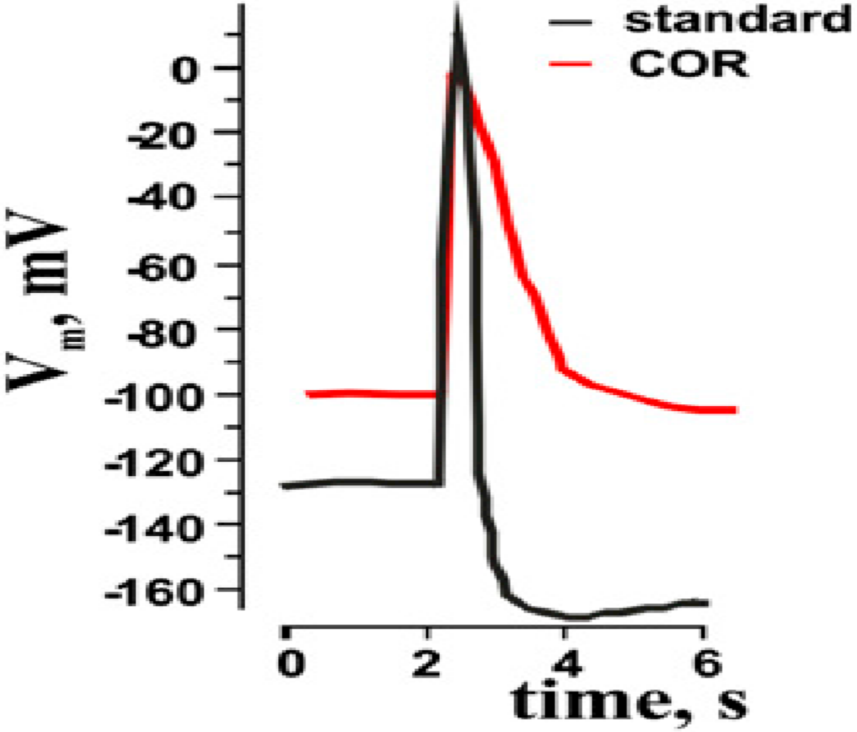
Venus Flytrap


+
Ag/AgCl
Electrodes
–
Venus Flytrap


i, μA
12
10
8
6
4
2
0
-2
-4
-6
-8
I / μA
0
v, V
-0.8
-0.6
-0.4
-0.2
0.0
0.2
0.4
V_P / V

Action Potential *Generated by* *calcium Ion-channel Memristors* *in the* *Venus Flytrap*





From : A special pair of phytohormones controls excitability, slow closure, and external stomach formation in the Venus flytrap

From : María Escalante-Pérez, Elzbieta Krol, Annette Stange, Dietmar Geiger, Khaled A. S. Al-Rasheid, Bettina Hause, Erwin Neher, and Rainer Hedrich

Proc. of *National Academy of Science*, *vol. 108*, *September 13, 2011*.

1991 Nobel Prize in Physiology

Discovers the function of single ion channels in cells

Erwin Neher

Voltage-Gated

Ion Channels

are

Memristor

Where there is

smoke

there is

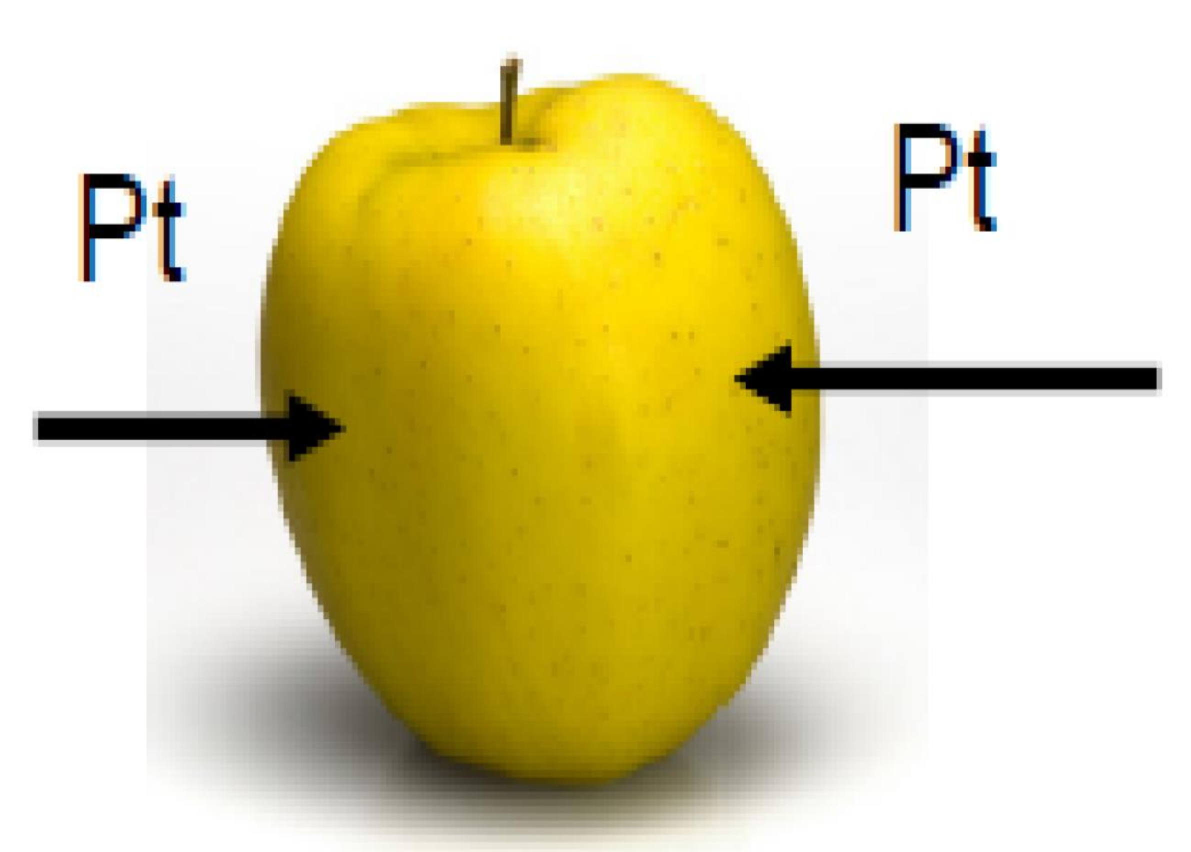
fire

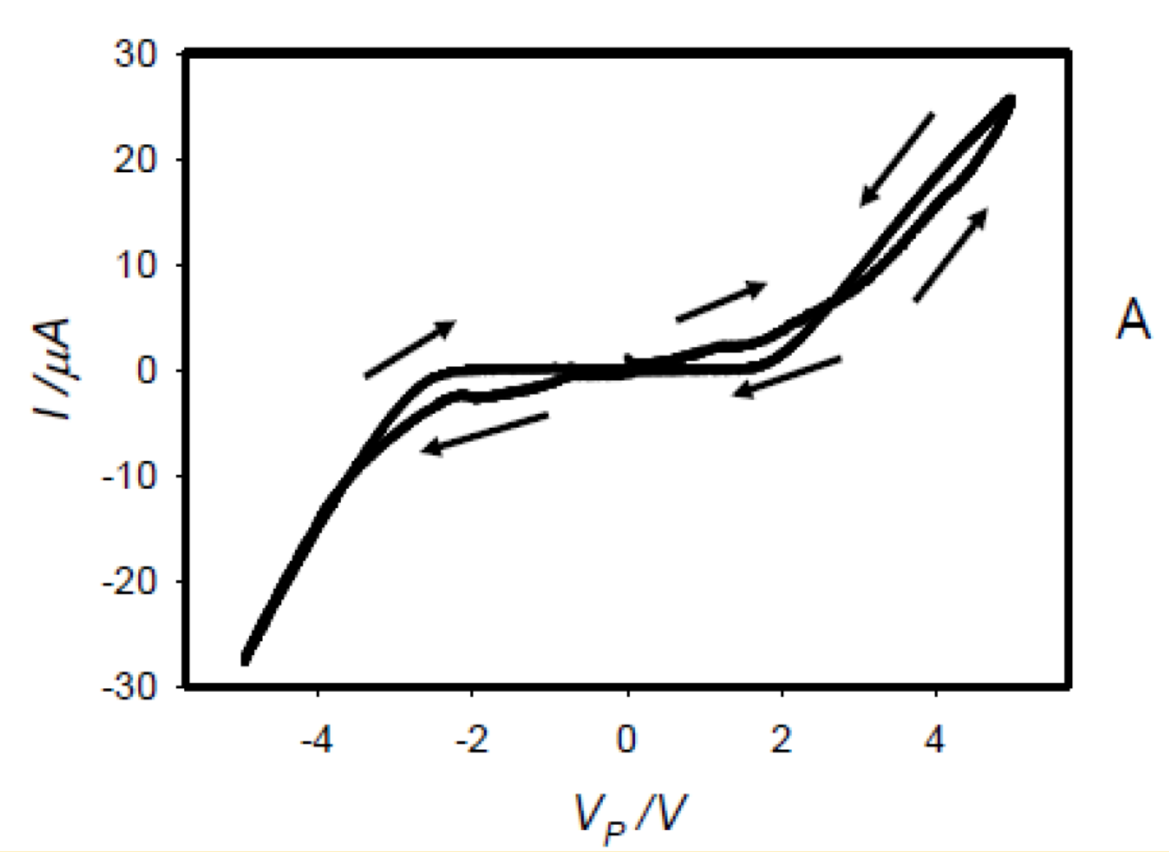
Where there is

ion

there is

memristor

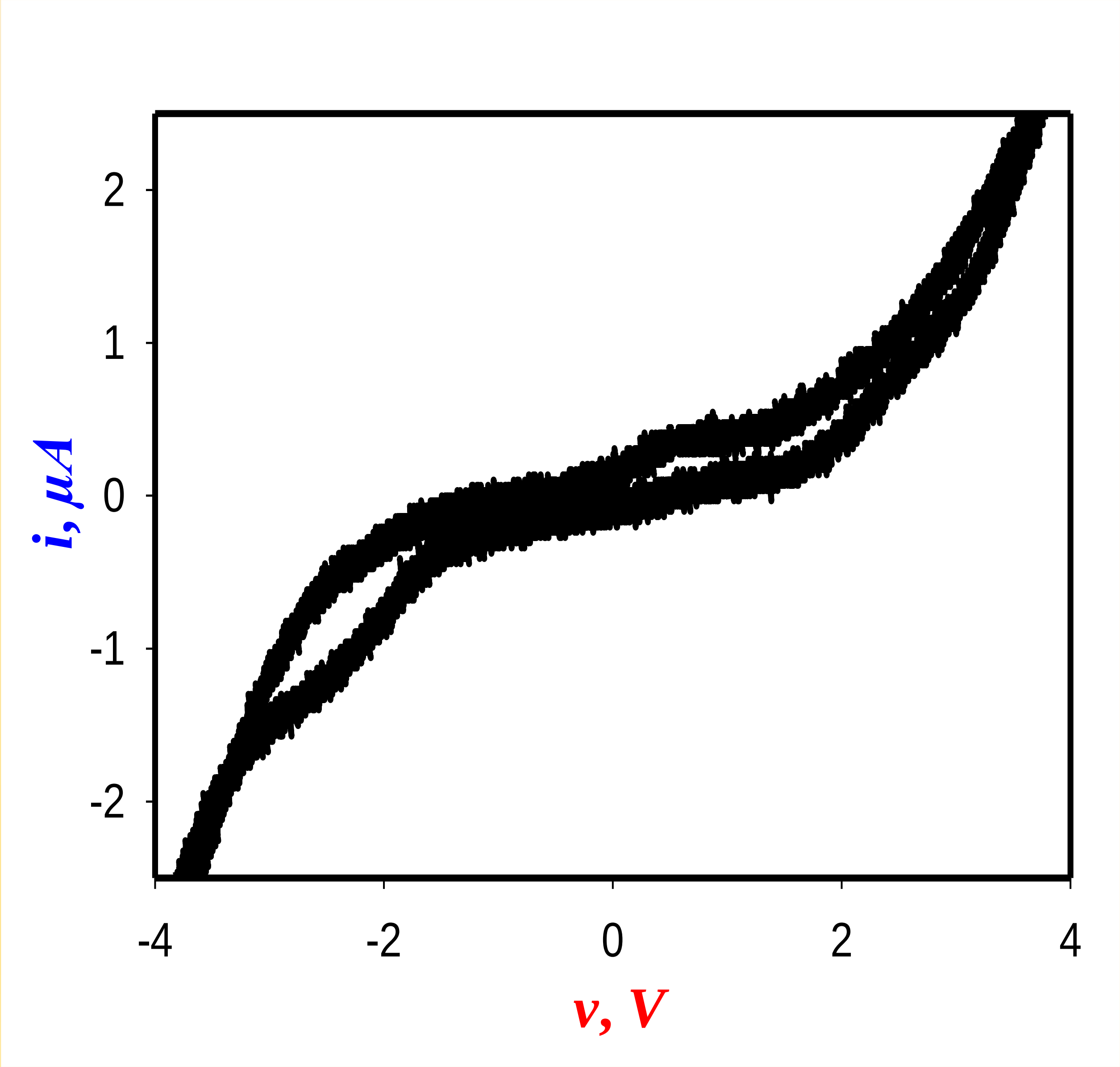

Pt
Pt


30
20
10
0
-10
-20
-30
I /μA
-4
-2
0
2
4
V_P /V
A

A SunPatiens Flower

Pt

Pt



Alessandro Volta

Volta explains the principle of the *"electrode column"* to Napoleon

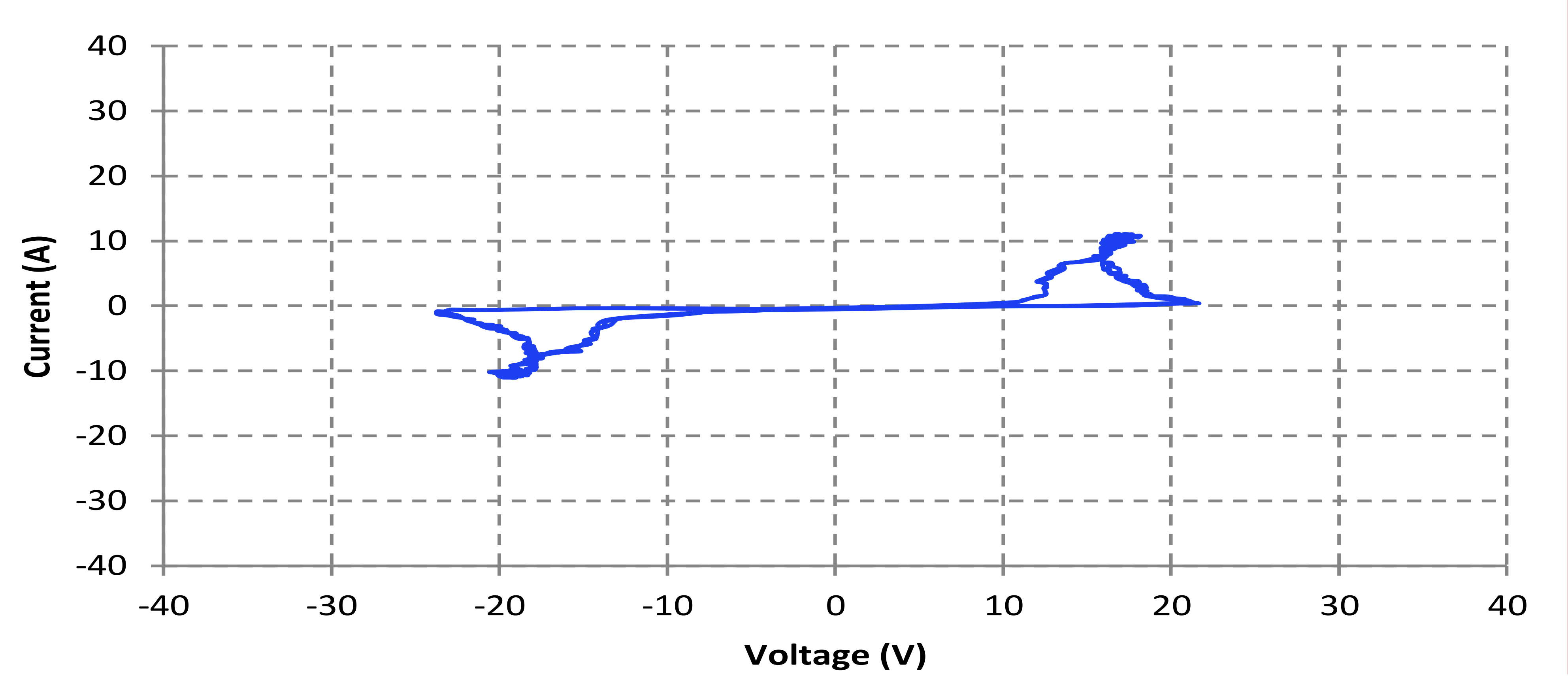
In honor of his invention Volta was made a *count* by Napoleon in **1801**.

Voltaic Pile
(Invented by Volta **1800**)

Setup of the carbon rod electrodes

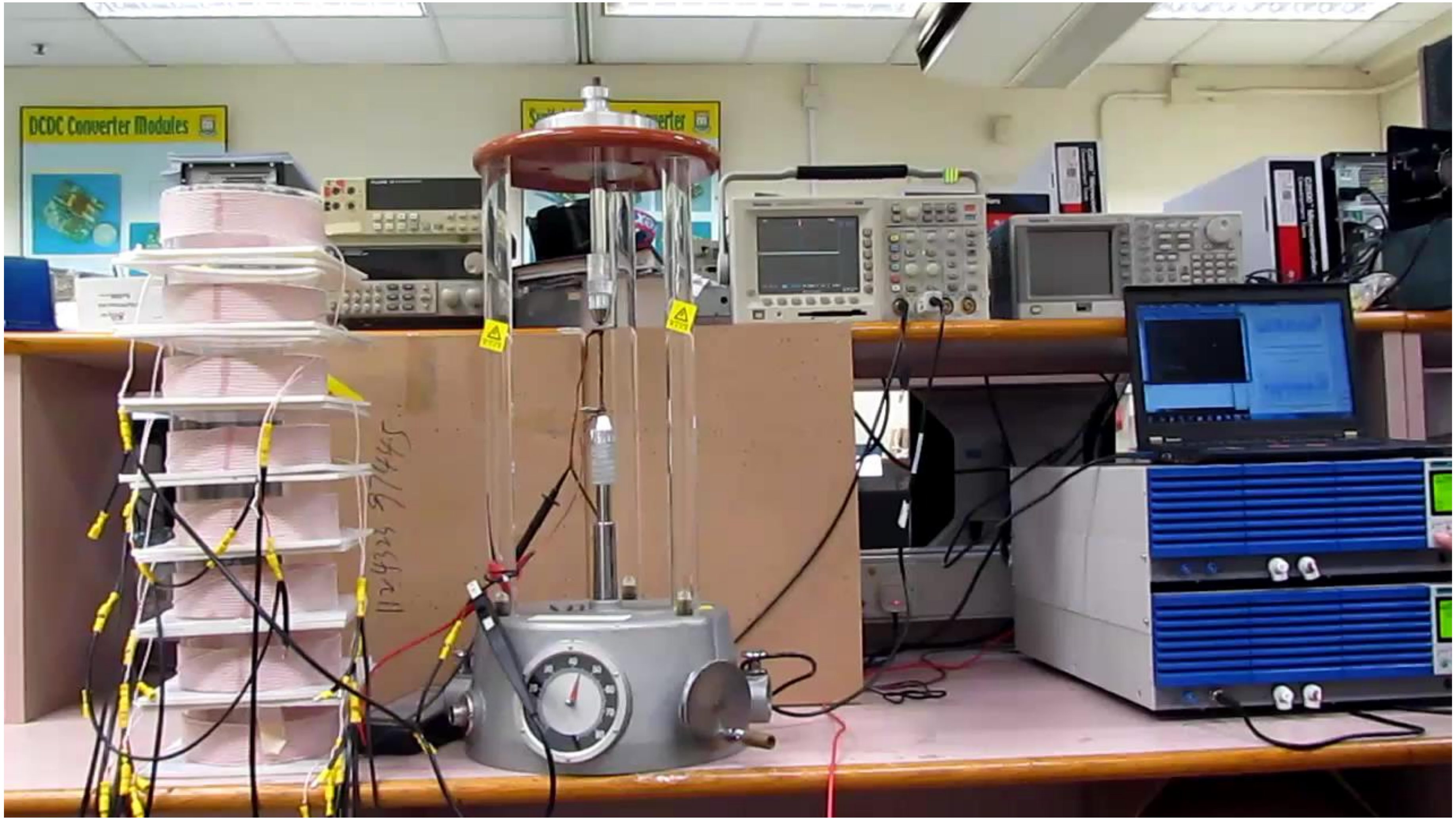
Carbon arc discharge in operation

Davy's Carbon-Rod Electrodes Exhibits the Memristor Pinched-Hysteresis Loop Fingerprints



Input Testing Signal: 3 KHz Sinusoidal Voltage

Video



Bonaparte Napoleon

Sir Humphry Davy

In 1808, when France was at war with England, ***Bonaparte Napoleon*** has decided to award ***Sir Humphry Davy***

The Prix Napoleon de Institut !

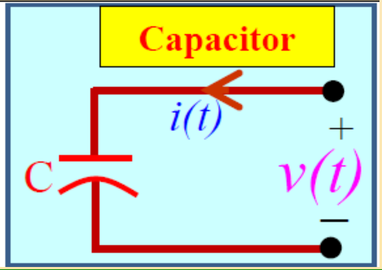
Faraday Invented the *Inductor* in 1831

Michael Faraday

4 Basic Circuit Elements

1745

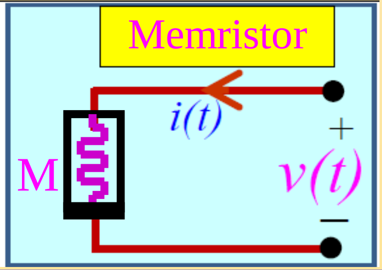
Ewald Georg Von Kleist



$$i(t) = C\frac{dv(t)}{dt}$$

1801

Sir Humphry Davy

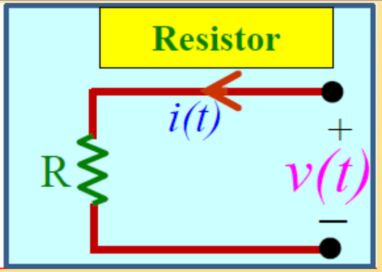


$$v(t) = R(x)\,i(t)$$

$$\frac{dx}{dt} = f(x\,,\,i)$$

1827

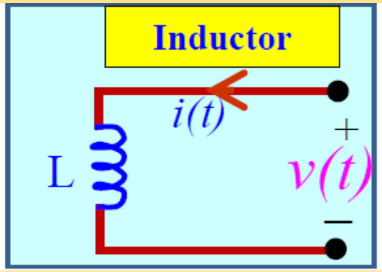
Georg Ohm



$$v(t) = R\,i(t)$$

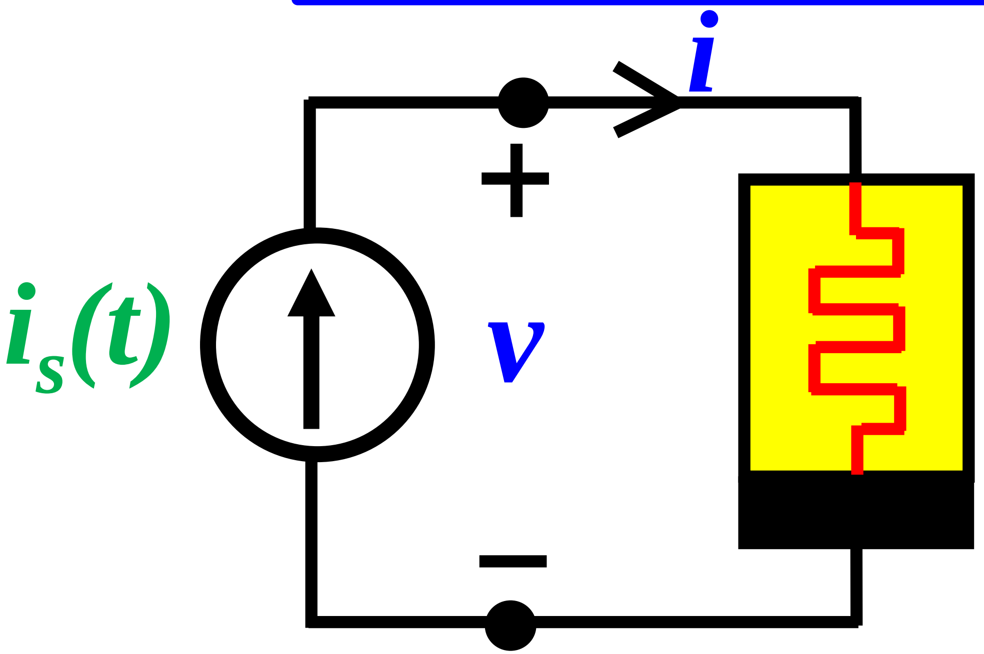
1831

Michael Faraday



$$v(t) = L\frac{di(t)}{dt}$$

The Most General *Memristors* are called *Extended Memristors*



Current-Controlled Memristor

State and Input-Dependent Ohm's Law

$$v = R(x, i)i$$

where

$$R(x, 0) \neq \infty$$

State Equation

$$\frac{dx}{dt} = f(x, i)$$

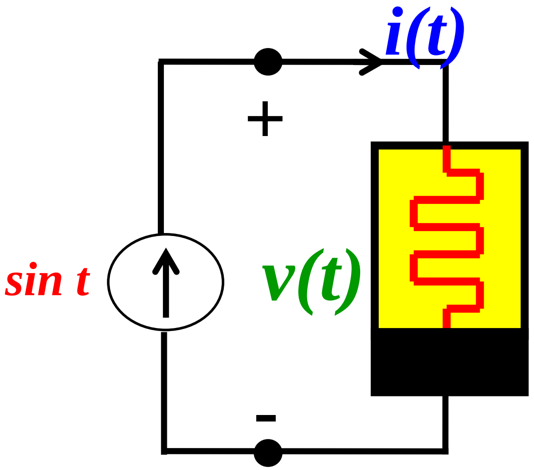
Why is the Condition

$$R(x,0) \neq \infty$$

necessary in the definition of the state and Current-Dependent Ohm's Law

$$v = R(x,i)\, i$$

An Extended Memristor with Unbounded Memristance



State-Dependent Ohm's Law

$$v = \underbrace{\left(\frac{x}{i}\right)}_{R(x,\,i)} i$$

State Equation

$$\frac{dx}{dt} = \underbrace{i}_{f(x,\,i)}$$

$$\boxed{i(t) = \sin t}$$

$$\frac{dx}{dt} = i = \sin t$$

Assuming $x(0) = -1$

$$x(t) = -1 + \int_0^t \sin t \, dt = -1 + \left[-\cos t + 1\right] = -\cos t$$

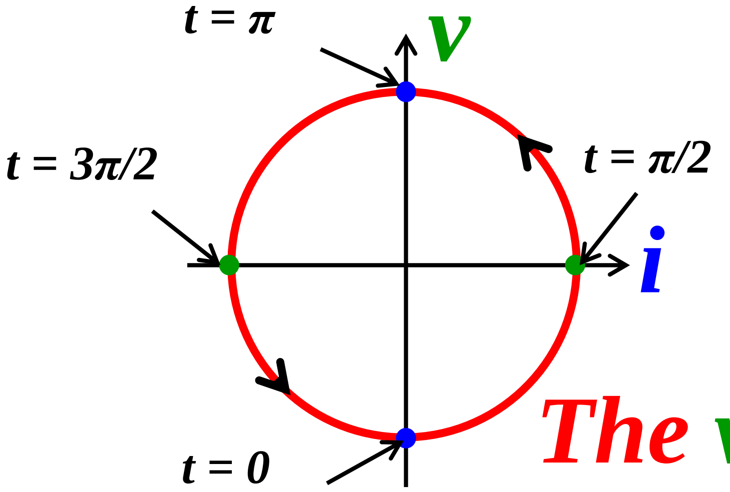
$$v(t) = \left[\frac{x(t)}{i(t)}\right] i(t) = x(t)$$

$$\boxed{v(t) = -\cos t}$$

Hence,

$$v^2(t) + i^2(t) = (-\cos t)^2 + (\sin t)^2 = 1$$

An Extended Memristor

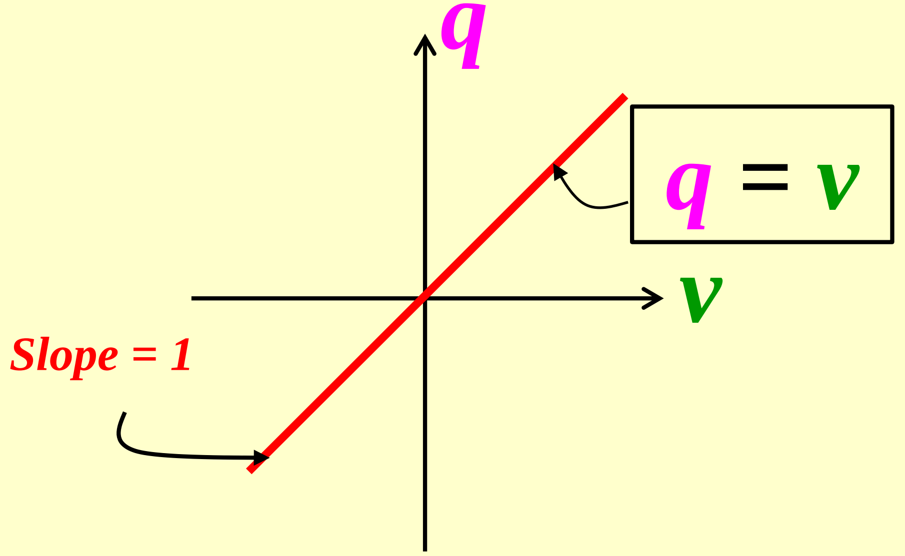


$$i = \sin t$$

$$v = -\cos t$$

$$v^2(t) + i^2(t) = 1$$

The v vs. i loci is NOT Pinched !



Observations :

(1). $v = \left(\frac{x}{i}\right) i = x$

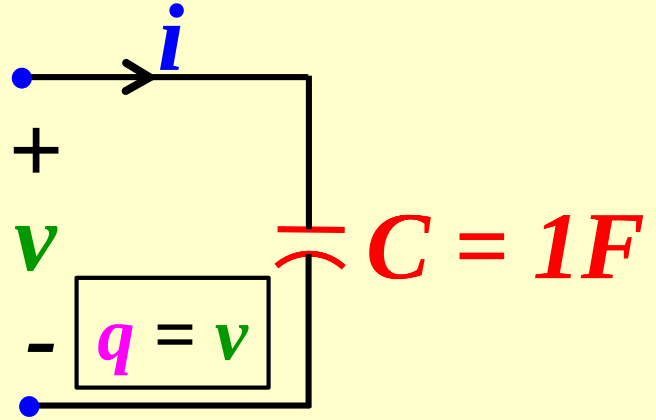
$R(x, i)$

(2). $\frac{dx}{dt} = i$

$f(x, i)$

$\therefore\ x = q$ (Charge)

(3). (1) and (2) $\Rightarrow$ $q = v$



If It's Not Pinched
It's Not A
MEMRISTOR

Example of An *Erroneous Memcapacitor*

PHYSICAL REVIEW B **81**, 195430 (2010)

Solid-state memcapacitive system with negative and diverging capacitance

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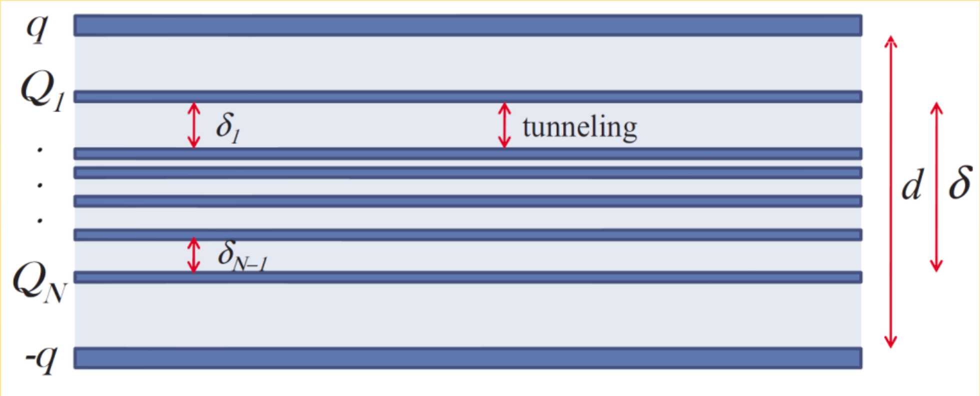
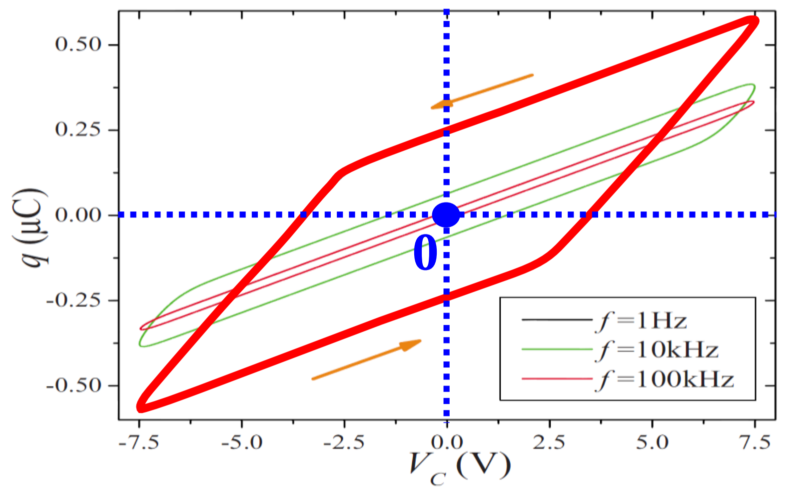
(Received 24 February 2010; revised manuscript received 30 April 2010; published 25 May 2010)

We suggest a possible realization of a solid-state memory capacitive (memcapacitive) system. Our approach relies on the slow polarization rate of a medium between plates of a regular capacitor. To achieve this goal, we consider a multilayer structure embedded in a capacitor. The multilayer structure is formed by metallic layers separated by an insulator so that nonlinear electronic transport (tunneling) between the layers can occur. The suggested memcapacitor shows hysteretic charge-voltage and capacitance-voltage curves, and both negative and diverging capacitance within certain ranges of the field. This proposal can be easily realized experimentally and indicates the possibility of information storage in memcapacitive systems.



General scheme of a solid-state memcapacitor. A metamaterial medium consisting of *N* metal layers embedded into an insulator is inserted between the plates of a "regular" capacitor.



Charge-voltage plot at different applied voltage frequencies *f*. The decrease in the hysteresis at higher frequencies is a signature of memcapacitors.

Hysteresis loop in *q* - *v* plane is ***not pinched***

➡ **not a *memcapacitor***